Comments on QCD Factorization


Matthias Neubert
Cornell University

March 17, 2005

CKM Workshop @ San Diego
- Convenors did most of the job by providing a detailed list of questions and tasks
- Will follow as closely as is reasonable …
Assumptions

- $m_b \gg \Lambda_{\text{QCD}}$ (heavy-quark limit)
- QCDF is the leading term in a power expansion in $\Lambda_{\text{QCD}}/m_b$ (all orders in $\alpha_s$)
  
  - Like any other systematic approach to B physics!
- Heavy-to-light form factors are treated as $O(1)$ nonperturbative objects (existence of soft overlap contribution proved in SCET)

Assumptions

- Perturbation theory is used at hard scale $m_b$ and hard-collinear scale $(2E_M \Lambda_{QCD})^{1/2}$
  - Good convergence checked in higher-order calculations
  - Perturbation theory at hard-collinear scale is needed for any determination of $V_{ub}$
  - Similar scale relevant in $\tau$ physics
- Note: no power corrections $\sim (\Lambda_{QCD}/m_b)^{1/2}$
Practical limitations

- Many input parameters (unavoidable – complicated physics!)
  - Form factors ($F_{B \to \pi}$), light-cone distribution amplitudes ($\Phi_{\pi}$), hadronic parameters ($\lambda_B$), quark masses ($m_s, m_c$)
  - $\Lambda_{QCD}/m_b$ power corrections can only be modeled (unavoidable, since factorization fails beyond leading power)
  - How important?
Practical limitations

- Presence of small external parameters complicates heavy-quark expansion
  - Correction $\sim \alpha_s C_1$ or $C_1/m_b$ can easily compete with leading-order term $\sim C_2$
  - Correction $\sim \alpha_s$ or $1/m_b$ can easily compete with leading-order term $\sim \lambda^2$
  - Must be taken into account when considering convergence of heavy-quark expansion
- Potentially important for ratios like $C/T$
Size of corrections

- Naively expect $\Lambda_{\text{QCD}}/m_b \sim 10\%$, but corrections are often larger (see above)
- Important example of sizeable power corrections: scalar penguin, weak annihilation also known as “charming penguins” 😊
Neglected amplitudes

- None (in principle)!
Successes of QCD factorization

- Have not done a fit to data since 2001
- Have not done an update on QCDF since 2003 (but see work by CKM Fitter group)
- Instead, look at scenario S4 in B&N, which gave a reasonable description of the data in 2003

$\mathcal{B}(B \rightarrow K\pi, \pi\pi, KK)$

CLEO
Belle
BABAR
PDG2002
New Avg.

HFAG
AUGUST 25th 2004

Branching Ratio x 10^6
$B(B \to K\pi, \pi\pi, KK)$

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Branching Ratio $\times 10^6$
$B(B \to (\eta, \eta') (K^\ast, \pi, \rho))$  

$B(B \to (K^\ast, \rho, \omega, \phi)(\pi, K, \eta, \eta'))$  

![Diagram showing branching ratios for various particle decays involving $\eta, \eta', K, \pi, \rho, \omega, \phi$.](image-url)
CP Asymmetry in Charmless B Decays

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CP Asymmetry in Charmless B Decays

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Successes of QCD factorization

- Consistent global description of a vast variety of hadronic and radiative two-body decays, with branching fractions ranging from $10^{-4}$ to $10^{-9}$
- Dynamical explanation of smallness of direct CP asymmetries (syst. cancellation of FSI phases in heavy-quark limit)
Successes of QCD factorization

- Dynamical explanation of huge $B \rightarrow \eta' K$ branching fractions
- Dynamical explanation of smallness of $P_V/P_P$ amplitude ratios ($B \rightarrow PV$ and $VP$)
- Dynamical explanation of intricate pattern of penguin interference seen in $B \rightarrow PP$, $B \rightarrow PV$, $B \rightarrow VP$ modes (see below)
Successes of QCD factorization

Fitting amplitudes is nice, understanding the strong dynamics that explains them is exciting!
Failures of QCD factorization

- None, really…
- But:
  - Evidence exists for certain sizeable power corrections, assuming no New Physics (should not be surprising)
  - Color-suppressed tree amplitudes (e.g., $B \rightarrow \pi \pi$), weak annihilation amplitudes (e.g., $B \rightarrow \pi^+ K^-$ CP asymmetry)
Results for specific amplitudes

- $B \to \pi\pi\pi$ tree amplitude (T+C):

$$10^6 \text{Br}(B^- \to \pi^-\pi^0) = (6.1^{+1.1}_{-0.7}) \times \left[ \frac{|V_{ub}|}{0.0037} \frac{F_{0}^{B \to \pi}(0)}{0.28} \right]^2$$

- Extract:

$$|V_{ub}| F_{B \to \pi}(0) = (0.98 \pm 0.05_{\text{exp}} \pm 0.07_{\text{th}}) \cdot 0.004 \cdot 0.25$$
Results for specific amplitudes

- Clean extraction of (T+C) possible using semileptonic decay:

\[
\frac{\Gamma(B^- \rightarrow \pi^- \pi^0)}{d\Gamma(\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu})/dq^2|_{q^2=0}} = 3\pi^2 f_\pi^2 |V_{ud}|^2 |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2
\]

- Important measurement!
Results for specific amplitudes

- Ratio of color-suppressed to color-allowed tree amplitude in $B \rightarrow \pi \pi$:

<table>
<thead>
<tr>
<th></th>
<th>Default</th>
<th>S2</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1(\pi \pi)$</td>
<td>$0.99^{+0.04}_{-0.07}$</td>
<td>0.84</td>
<td>0.88</td>
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<tr>
<td>$\alpha_1(\pi \rho)$</td>
<td>$0.99^{+0.04}_{-0.06}$</td>
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<td>0.90</td>
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<tr>
<td>$\alpha_1(\rho \pi)$</td>
<td>$1.01^{+0.04}_{-0.05}$</td>
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<td>$\alpha_2(\pi \pi)$</td>
<td>$0.20^{+0.17}_{-0.11}$</td>
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<td>$\alpha_2(\pi \rho)$</td>
<td>$0.20^{+0.16}_{-0.10}$</td>
<td>0.45</td>
<td>0.41</td>
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<tr>
<td>$\alpha_2(\rho \pi)$</td>
<td>$0.16^{+0.13}_{-0.09}$</td>
<td>0.31</td>
<td>0.30</td>
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</tbody>
</table>

Ratio $\alpha_2/\alpha_1$ equals $C/T \sim 0.3 – 0.7$
Results for specific amplitudes

- Clean extraction of penguin amplitudes in $B^{\pm} \rightarrow \pi^{\pm} K^0$, $B^{\pm} \rightarrow \pi^{\pm} K^*0$, $B^{\pm} \rightarrow \rho^{\pm} K^0$:

\[
\left| \frac{\hat{\alpha}_4^c(\pi \bar{K})}{\alpha_1(\pi \pi) + \alpha_2(\pi \pi)} \right| = \left| \frac{V_{ub}}{V_{cb}} \right| \left| \frac{f_\pi}{f_K} \right| \left[ \frac{\Gamma(B^- \rightarrow \pi^- \bar{K}^0)}{2\Gamma(B^- \rightarrow \pi^- \pi^0)} \right]^{1/2}
\]

- Important lessons about interference of different penguin amplitudes (scalar vs. vector penguins)
Anatomy of penguins

\[ B^\pm \rightarrow \pi^\pm K^0 \]

\[ B^\pm \rightarrow \pi^\pm K^{*0} \]
Results for specific amplitudes

- **Tree-to-penguin ratios:**

<table>
<thead>
<tr>
<th></th>
<th>Theory</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>T_{\pi\pi}</td>
<td>$</td>
<td>$0.91^{+0.05}_{-0.07}$</td>
<td>0.75</td>
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<tr>
<td>$</td>
<td>P_{\pi\pi}/T_{\pi\pi}</td>
<td>$</td>
<td>$0.32^{+0.16}_{-0.09}$</td>
<td>0.49</td>
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<tr>
<td>$</td>
<td>T_{\pi\rho}</td>
<td>$</td>
<td>$0.98^{+0.04}_{-0.07}$</td>
<td>0.88</td>
</tr>
<tr>
<td>$</td>
<td>P_{\pi\rho}/T_{\pi\rho}</td>
<td>$</td>
<td>$0.10^{+0.06}_{-0.04}$</td>
<td>0.12</td>
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<tr>
<td>$</td>
<td>T_{\rho\pi}</td>
<td>$</td>
<td>$1.07^{+0.09}_{-0.07}$</td>
<td>1.03</td>
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<tr>
<td>$</td>
<td>P_{\rho\pi}/T_{\rho\pi}</td>
<td>$</td>
<td>$0.10^{+0.09}_{-0.05}$</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Results for specific amplitudes

- Smallness of penguins in PV and VP modes understood in terms of intricate interference of penguin contributions from (V-A)(V±A) operators ($P_4$) and (S-P)(S+P) operators ($P_6$), called $a_4$ and $a_6$ in QCD factorization.

- Note that $a_4 \approx 1$ whereas $a_6 \approx 1/m_b$ (formally power suppressed, but “chirally enhanced”), but both are predicted to have similar size in QCD factorization.
Results for specific amplitudes

- Interference pattern:
  - $B \rightarrow PP$: $P_4 + P_6$
  - $B \rightarrow PV$: $P_4 + \varepsilon P_6$  $\leftarrow$ dynamical suppression of $P_6$
  - $B \rightarrow VP$: $P_4 - P_6$  $\leftarrow$ destructive interference
  - $B \rightarrow VV$: combination of PV, VP cases

- Data confirm this pattern, which was predicted by QCD factorization

- Clear evidence for power corrections to penguin amplitudes (neglected in “SCET approach”)
Important future work

- Calculate BBNS kernels at 2-loop order
  - May well be important (evidence for significant corrections from BLM terms)
- More systematic study of power corrections, both in theory (SCET) and phenomenology
- Perform new, comprehensive analysis of all available data
Comments on SCET approach

Comments on SCET approach

- SCET is an effective field-theory framework for studying factorization properties of decay amplitudes (useful for factorization proofs)
- It is **not** the basis of a new (or improved) factorization approach
- “SCET approach” of Bauer, Pirjol, Rothstein, Stewart is a different way of applying the QCD factorization formula
Comments on SCET approach

Assumptions:

- Charm penguin *may* not be factorizable, so fit it to data
- Perturbation theory at hard-collinear scale \((m_b\Lambda_{QCD})^{1/2}\) *may* be in trouble, so do not use it
- Set all power corrections to zero
- Set all perturbative corrections to zero (since otherwise predictive power is lost!)
Comments on SCET approach

- Rather restrictive set of assumptions, not supported by theory
- In particular, neglect of $\alpha_s$ corrections means that theoretical accuracy is same as in naïve factorization
- Method should be called BPRS approach
Comparison of approaches

<table>
<thead>
<tr>
<th></th>
<th>Expansion parameters</th>
<th>Predictive power</th>
<th>1/m_b corrections</th>
<th>Global predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCDF</td>
<td>( \Lambda/m_b, \alpha_s(m_b) ) ( \alpha_s(m_b/\Lambda) )</td>
<td>LO in 1/m_b, all orders in ( \alpha_s )</td>
<td>modeled (parts calculated)</td>
<td>all B( \rightarrow ) PP, PV, VP modes</td>
</tr>
<tr>
<td>pQCD</td>
<td>( \Lambda/m_b, \alpha_s(m_b) ) ( \alpha_s(m_b/\Lambda) )</td>
<td>all orders in ( \alpha_s )</td>
<td>modeled (parts calculated)</td>
<td>many modes</td>
</tr>
<tr>
<td>BPRS</td>
<td>( \Lambda/m_b, \alpha_s(m_b) )</td>
<td>LO in 1/m_b (excl. charm penguin), tree level in ( \alpha_s )</td>
<td>ignored</td>
<td>B( \rightarrow ) ( \pi \pi \pi )</td>
</tr>
</tbody>
</table>