CP Violation in $B \rightarrow \rho \rho$ and Measurement of the Unitarity Triangle Angle $\alpha$

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Adrian Bevan
Talk Overview

• Physics Motivation
  • B→VV decays (quick reminder)
  • CP Violation (v. quick reminder)
  • Isospin analysis (even quicker reminder)

• Results
  • $\rho^0\rho^0$ Result from ICHEP (210fb$^{-1}$)
  • $\rho^+\rho^-$ (210fb$^{-1}$)
  • a few words on $\rho^+\rho^0$

• Impact on the Unitarity Triangle

• Conclusions
$B \rightarrow \rho\rho$

- VV final state
- $L=0,1,2$ partial wave components – need angular analysis to determine CP content of decay
- $\rho\rho$ is almost 100% longitudinally polarised ($f_L \sim 1$)

\[
\frac{d^2\Gamma}{\Gamma d \cos \theta_1 d \cos \theta_2} = \frac{9}{4} \left( f_L \cos^2 \theta_1 \cos^2 \theta_2 + \frac{1}{4} (1 - f_L) \sin^2 \theta_1 \sin^2 \theta_2 \right)
\]

Longitudinal (CP even)  
SLong & CLong

Transverse (Mixed CP state)  
Set $S_{\text{Tran}} = C_{\text{Tran}} = 0$  
& vary for systematics
Like $\pi\pi$ etc., analyse time evolution of $B^0\bar{B}^0$ system

$$f(B^0 / B^0_{phys} \rightarrow f / f^*, \Delta t) = \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} \left[1 + \eta S \sin(\Delta m_d \Delta t) - \eta C \cos(\Delta m_d \Delta t)\right]$$

(assume $\Delta \Gamma = 0$):

Indirect CP violation $\Rightarrow S \neq 0$

Direct CP violation $\Rightarrow C \neq 0$

$$\lambda_{f_{CP}} = \frac{q}{p} \frac{\bar{A}}{A_{f_{CP}}}$$

$$S_{f_{CP}} = \frac{2 \text{ Im } \lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^2}$$

$$C_{f_{CP}} = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2}$$

Also need to worry about: tagging dilution : resolution function

$BaBar : Belle$
But it’s a bit more complicated for $\rho\rho$...

In general want to do a time dependent angular analysis ($D^*D^*$).

As $f_L \sim 1$, most information is in a single CP even amplitude.

$\Delta t$ dependence is:

$$P_{LONG} \propto \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} \left[ 1 + \eta S_{LONG} \sin(\Delta m_d \Delta t) - \eta C_{LONG} \cos(\Delta m_d \Delta t) \right]$$

$$P_{TRAN} \propto \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} \left[ 1 + \eta S_{TRAN} \sin(\Delta m_d \Delta t) - \eta C_{TRAN} \cos(\Delta m_d \Delta t) \right]$$

Don’t care about disentangling CP admixture of the transverse polarisation into CP eigen states.

So as there are very few transverse events just set $S_{TRAN} = C_{TRAN} = 0$ in fit to data vary between ±1 when evaluating systematic error.
CP Violation in $B^0 \rightarrow \rho^+\rho^-$

**Tree Level:**

$V_{td} : \beta$

$V^*_{td} : \beta$

$C_{\rho\rho} = 0$

$S_{\rho\rho} = \sin(2\alpha)$

→ Same Physics as $\pi\pi/\rho\pi$

→ measure $\alpha_{\text{eff}}$

→ need to bound $|\alpha_{\text{eff}} - \alpha|$ (shift from loops)

$+\text{Loops (penguins)}$

$C_{\rho\rho} \propto \sin(\delta)$

$S_{\rho\rho} = \sqrt{1 - C_{\rho\rho}^2} \sin(2\alpha_{\text{eff}})$

$\delta = \delta_p - \delta_T$
Isospin Analysis

- Bounds $\alpha_{\text{eff}} - \alpha$

- SU(2) relates different $\rho \rho$ final states
  $\Rightarrow$ triangles for $\rho \rho$

\[
\frac{1}{\sqrt{2}} A_{\text{Long}}^{+-} + A_{\text{Long}}^{00} = A_{\text{Long}}^{+0}
\]

\[
\frac{1}{\sqrt{2}} A_{\text{Long}}^{--} + A_{\text{Long}}^{00} = A_{\text{Long}}^{--0}
\]

similar relations for each transversity amplitude
$\Rightarrow$ only care about longitudinal events as $f_L \sim 1$

- So if $A^{00} = 0$, we would expect $|A^{+-}|^2 = 2|A^{+0}|^2$

Isospin Analysis/determination of $\alpha$

$$\kappa_{\rho\rho} = 2 |\alpha_{\text{eff}} - \alpha|$$

For $\rho\rho$ we require:

- $A(B^0 \to \rho^+ \rho^-) + \text{C.C.}$
- $A(B^0 \to \rho^0 \rho^0) + \text{C.C.}$
- $A(B^+ \to \rho^+ \rho^0) + \text{C.C.}$
- $S_{\rho^+ \rho^-}$
- + Polarization: $f_L^{+-}, f_L^{+0}, f_L^{00}$

$$\alpha_{\text{eff}} = \frac{1}{2} \arcsin \left( \frac{S_{+-}}{\sqrt{1-C_{+-}^2}} \right) \rightarrow \alpha$$
$B^0 \rightarrow \rho^0 \rho^0$

Limiting factor in the $\rho\rho$ isospin analysis

First result published in PRL 91, 171802 (2003)

$BF(B^0 \rightarrow \rho^0 \rho^0) < 1.1 \times 10^{-6}$ (90% CL)

$N_{signal} = 33^{+22}_{-20} (stat)$

ML fit using 8 inputs:
- $m_{ES}$, $\Delta E$, tagging,
- $m_{\pi^+\pi^-}$, $\cos \theta_H$, MVA

$\times 2$

Dominant Systematics:
- $a_1\pi$ interference: $\pm 7.5$ events
- PDF variations: $\pm 6$ events
- $B$ bkgd: $\pm 5.8$ events

hep-ex/0412067 (accepted for publication)
Previous $\rho^+\rho^-$ results from BaBar

• Runs 1+2 (results published in PRL, PRD):

\[
BR(B^0 \rightarrow \rho^+ \rho^-) = (30 \pm 4 \pm 5) \times 10^{-6}
\]

\[
f_L = 0.99 \pm 0.03^{+0.04}_{-0.03}
\]

\[
S_{\text{Long}} = -0.42 \pm 0.41 \pm 0.14 \quad C_{\text{Long}} = -0.17 \pm 0.27 \pm 0.14
\]

• Runs 1+2+3 (presented at Moriond EW 2004):

\[
S_{\text{Long}} = -0.19 \pm 0.33 \pm 0.11 \quad C_{\text{Long}} = -0.23 \pm 0.24 \pm 0.14
\]
Using Grossman Quinn Bound to constrain penguin pollution

\[ \sin^2 \delta \leq \frac{f_{Long}^0 \times \mathcal{B}(B^0 / \bar{B}^0 \to \rho^0 \rho^0)}{f_{Long}^{\pm} \times \mathcal{B}(B^\pm \to \rho^\pm \rho^0)} \]

Isospin Analysis

\[ \alpha = 102^{+16^0}_{-12^0} \text{stat} \pm 13^\circ \text{penguin} \]

\[ \alpha = 96^\circ \pm 10^\circ \text{stat} \pm 4^\circ \text{syst} \pm 13^\circ \text{penguin} \]
New $\rho^+\rho^-$ Result

- Essentially same method; but there are a few changes

- 210 fb$^{-1}$ of data used (BaBar’s full data set)

- CP fit still uses untagged events to constrain $q\bar{q}$ background shape from data

- new result is an update of $S_{\text{Long}}$, $C_{\text{Long}}$, and $f_L$

- some improvements in the analysis
  - $\Delta t$ model for signal is improved
  - better B-background model (updated according to HFAG)
  - interference calculation for systematic errors

- use toy MC to extract CL on $\alpha$
• Unbinned extended maximum likelihood fit

\[
m_{ES} - B \text{ mass} \\
\Delta E - \text{energy difference} \\
\Delta t - \text{proper time difference} \\
\text{NN}_{out} - \text{neural network} \\
m_{\pi\pi} (x2) - \rho \text{ masses} \\
\cos\theta_{hel} (x2) - \rho \text{ helicity}
\]

\[
\text{categories:}\quad 5 \text{ exclusive tagging} \\
- \text{Leptons} \\
- \text{High Purity Kaons} \\
- \text{Lower Purity Kaons} \\
- \text{Other} \\
- \text{Un-tagged}
\]

\[
\text{NN}_{out}: \quad 10 \text{ inputs to a multi layer perceptron to discriminate between} \\
\text{signal and } e^+e^- \to \text{continuum} \\
\sum_{ROE} |p_i^*| \quad \text{for neutral and charged} \\
\sum_{ROE} |p_i^*| \cos^2 \theta_i \quad \text{sums over ROE used} \\
\sum_{ROE} p_T \quad \text{separately in MVA}
\]

\[
\pi^0 \text{ decay angle (x2)} \\
|\cos| \text{ of angle between direction of B and Z axis} \\
|\cos| \text{ of angle between the B thrust and Z axis} \\
|\cos| \text{ of angle between the B thrust axis and thrust of ROE}
\]
Improvements in $\Delta t$ model for latest result

Signal is the sum of:

- true signal (51.8%)
- correct track SCF (34.6%)
- wrong track SCF (13.6%)

Wrong track SCF
- pick up one or all tracks from other $B$ meson
- leads to biased $S$ and $C$ for part of the signal

(Correct Tracks used to form vertex)
- calculating the correct $\Delta z/\Delta t$

(Wrong Track(s) used)
- wrong $\Delta z/\Delta t$: ‘junk’ for CP fit
**Solution**: describe $S$ and $C$ for correct track signal and wrong track signal separately.

\[
PDF_{signal} = PDF_{long}^{signal} + PDF_{tran}^{signal} \\
PDF_{long}^{signal} = (P_{true}^{long} + P_{correct\ track}^{long}) \times f[S_{Long}, C_{Long}, \Delta t, \sigma_{\Delta t}] \\
+ P_{wrong\ track}^{long} \times f[S_{Long}^{W}, C_{Long}^{W}, \Delta t, \sigma_{\Delta t}]
\]

- Use the correct track signal only for measuring $S_{LONG}$ and $C_{LONG}$.
- Set $S_{Long}^{w}=C_{Long}^{w}=0$ in the fit & apply systematic error for wrong track CP on the final result.

**Benefits of this solution**
- remove bias on CP result due to these events (~1.3° effect).
- no need for additional systematic errors to cover this.
Brief Summary of Time Dependence of fit

• split the longitudinal and transverse polarisation

• longitudinal signal:
  • correct track S and C are the $S_{\text{Long}}$ and $C_{\text{Long}}$ of our result. We use this for $\alpha$
  
  • wrong track S and C are set to zero in the nominal fit. Then vary these parameters to $\pm 1$ for systematic errors

• transverse signal:
  • do not distinguish between CP odd/even components. Just allow for a common S and C. Vary between $\pm 1$ for systematics

• Correct track events have a common resolution function

• Wrong track events have their own resolution function
Background Summary

In addition to the signal (1% of fit sample)
→ continuum background (92% of fit sample)
→ B background 7% of fit sample

The Main B backgrounds Considered In the fit (38 different modes are included in our background model):

**B → charm dominates**

\[
\begin{align*}
B^+ &\rightarrow charm = 2551 \\
B^+ &\rightarrow (a_1\pi)^+ = 87 \\
B^+ &\rightarrow \rho^+\rho^0 = 82 \\
B^+ &\rightarrow a_1\rho^+ = 65 \\
B^+ &\rightarrow \rho\pi^0 = 51 \\
B^0 &\rightarrow charm = 1316 \\
B^0 &\rightarrow a_1\rho = 145 \\
B^0 &\rightarrow a_1\pi = 65 \\
B^0 &\rightarrow K^{**}\pi = 56 \\
B^0 &\rightarrow \rho\pi = 31
\end{align*}
\]

Final states with charged ρ mesons contribute a lot in our background model.
Fit Results

A total of 26 parameters are floated.

- signal yield
- $S_L, C_L, f_L$
- 22 background parameters

Data sample = 68703 events

\[
N_{signal} = 685 \pm 52 \quad \rightarrow \quad N_{signal} = 617 \pm 52
\]

\[
f_L = 0.978 \pm 0.014
\]

\[
S_L = -0.33 \pm 0.24
\]

\[
C_L = -0.03 \pm 0.18
\]

- Correct signal yield for bias coming from correlations that are neglected in the fit model
- These have a small impact on the other parameters (see systematics)
Signal Projection Plots – Lepton and Kaon 1 categories only

**Cut on nno**

- $\varepsilon_{\text{signal}} = 14\%$
- $\varepsilon_{bg} = 1.5\%$

**Cut on nno and mes**

- $\varepsilon_{\text{signal}} = 11.5\%$
- $\varepsilon_{bg} = 0.4\%$

**Cut on nno and mes**

- $\varepsilon_{\text{signal}} = 11.5\%$
- $\varepsilon_{bg} = 0.4\%$

$\cos^2\theta$ distribution $\times$ acceptance for signal + asymmetric background

**Total likelihood**

- Events / (4 MeV/c$^2$)

**Total background**

- Events / (18 MeV)

- Events / (50 MeV)

**$m_{ES}$ (GeV/c$^2$)**

- $\Delta E$ (GeV)

- $m_{\pi\pi}$ (GeV/c$^2$)
Interference calculation (systematic error)

1. Generate toy MC in 5D phase space: \( m_1, m_2, \theta_1, \theta_2, \phi \) for \( A_{\rho \rho} \) and \( A_{xx} \).
   
   \[ xx = a_1 \pi, \, \rho \pi \pi, \, \pi \pi \pi \pi \] using BW for resonances

2. Calculate Matrix element including interference term

3. Cut on allowed \( m_i \) and \( \theta_i \) to match \( \rho \rho \) selection cuts

4. Calculate effective S and C from toy data: e.g.
   
   \[
   \hat{\lambda}(x_i) = \frac{q}{p} \left( \frac{A(\overline{B}^0 \rightarrow \rho^+ \rho^-) + A(\overline{B}^0 \rightarrow a_1^+ \pi^-) + A(\overline{B}^0 \rightarrow a_1^- \pi^+)}{A(B^0 \rightarrow \rho^+ \rho^-) + A(B^0 \rightarrow a_1^+ \pi^-) + A(B^0 \rightarrow a_1^- \pi^+)} \right) \Rightarrow S(x_i) = \frac{2 \text{Im}(\hat{\lambda}(x_i))}{1 - |\hat{\lambda}(x_i)|^2}
   \]

   \[
   S^{\text{eff}}(x_i) = \frac{\int S(x_i) \left| A(\overline{B}^0 / B^0 \rightarrow a_1^+ \pi^-) \right|^2 d^5 x_i}{\int \left| A(\overline{B}^0 / B^0 \rightarrow a_1^+ \pi^-) \right|^2 d^5 x_i}
   \]

5. Take the RMS deviation of \( O_{\text{eff}} - O \) as the systematic error.

   • also looked at \( \sigma(400) \pi \pi \), but ignored this final state as it has a small efficiency relative to the other final states included here.
## Total Systematic Uncertainty

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$f_L$</th>
<th>$S_{long}$</th>
<th>$C_{long}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$-background yield</td>
<td>±0.003</td>
<td>±0.027</td>
<td>±0.002</td>
</tr>
<tr>
<td>$B$-background $CP$</td>
<td>±0.001</td>
<td>±0.027</td>
<td>±0.045</td>
</tr>
<tr>
<td>non-resonant events</td>
<td>0.015</td>
<td>0.030</td>
<td>0.001</td>
</tr>
<tr>
<td>Floating $B$- background</td>
<td>-0.020</td>
<td>-0.12</td>
<td>+0.008</td>
</tr>
<tr>
<td>Neglecting Interference</td>
<td>0.0036</td>
<td>0.023</td>
<td>0.022</td>
</tr>
<tr>
<td>DCSD</td>
<td>–</td>
<td>0.012</td>
<td>0.037</td>
</tr>
<tr>
<td>SVT LA (alignment)</td>
<td>–</td>
<td>0.034</td>
<td>0.005</td>
</tr>
<tr>
<td>Fit bias</td>
<td>0.010</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>CPV in wrong track SXF</td>
<td>0.0005</td>
<td>+0.007</td>
<td>±0.012</td>
</tr>
<tr>
<td>Total</td>
<td>±0.021</td>
<td>+0.08</td>
<td>±0.09</td>
</tr>
</tbody>
</table>

Table 1: Summary of the main systematic errors contribution to $f_L$, $S_{long}$ and $C_{long}$
Events from the Lepton and Kaon1 tagging categories only.

\[
S_{Long} = -0.33 \pm 0.24^{+0.08}_{-0.14}
\]

\[
C_{Long} = -0.03 \pm 0.18 \pm 0.09
\]

\[
f_L = 0.978 \pm 0.014^{+0.020}_{-0.028}
\]
UT Fit and CKM Fitter predict that the central value of BF(B⁺→ρ⁺ρ⁰) is a little too large for SU(2).

An updated measurement of this mode is long overdue & we’re eagerly awaiting it!
Measuring $\alpha$

Use Isospin analysis for longitudinal polarisation

$B^0 \to \rho^+ \rho^-$ (BaBar)

$$BF = (30 \pm 4 \pm 5) \times 10^{-6}$$

$$f_L = 0.978 \pm 0.014^{+0.021}_{-0.028}$$

$B^+ \to \rho^+ \rho^0$ (Belle & BaBar)

$$BF = (26.4 \pm 6.4) \times 10^{-6}$$

$$f_L = 0.96^{+0.05}_{-0.07}$$

$B^0 \to \rho^0 \rho^0$ (BaBar)

$$BF < 1.1 \times 10^{-6} (90\% C.L.)$$

Penguins are small in $B \to \rho \rho$

$$|\alpha_{\text{eff}} - \alpha| < 11^\circ (68\% CL)$$
• Build a $\chi^2_{\text{min}}$ from the measured observables (S,C, f_L and BR) each of which can be related to the amplitudes that make up the isospin triangles.

\[ \chi^2 \text{ scan in } \alpha \quad \Delta \chi^2 (\alpha) = \chi^2_{\text{min}} (\alpha) - \chi^2_{\text{min}0} \]

To obtain the CL: we use toy MC to reproduce what happens with data:  
  i) Generate toy for each alpha scanned  
  ii) calculate $1 - \text{CL}(\alpha)$: fraction of toys with  

\[ \Delta \chi^2 (\alpha)_{\text{toy}} > \Delta \chi^2 (\alpha)_{\text{data}} \]

Toy MC techniques work better in this case as the isospin triangle is not closed (so is unphysical):  

\[ 1 / \sqrt{2} \left| A_{\text{Long}}^{++} \right| + \left| A_{\text{Long}}^{00} \right| \leq \left| A_{\text{Long}}^+ \right| \]
Assumptions in the use of SU(2) with $\rho^+\rho^-$

- Assume Gronau-London Isospin analysis (use SU2 without electroweak penguins)

- Neglect I=1 component of amplitude (should be $O(\Gamma_\rho/m_\rho)^2 \approx 4\%$)

  - (implicitly we also neglect higher $\rho$ resonances in the final state when extracting S and C)
The $\alpha$ Result

The solution in agreement with the SM is:

$$\alpha = 100^\circ \pm 13^\circ$$

**Neglect:**
- EW penguins
  - $(1-2^\circ)$ effect (CKM Fitter/Gronau & Zupan)
- Possible $I=1$ amplitudes

**Inputs from other $\rho\rho$ measurements**

$$S_{Long} = -0.33 \pm 0.24_{-0.14}^{+0.08}$$
$$C_{Long} = -0.03 \pm 0.18 \pm 0.09$$
$$f_L = 0.978 \pm 0.014_{-0.028}^{+0.020}$$

$\alpha \in [79^\circ, 123^\circ]$ at $90\%CL$
• Using these results, we can constrain $|P/T|$ in $\rho\rho$
• and also show the $\alpha$ constrain in the $\eta\rho$ plane.

\[
P = P_t - P_c
\]
\[
T = T + P_u - P_c
\]

(same $P$, $T$ convention as CKM Fitter for the $|P/T|$ plot)
Summary

- New result on $\rho^0\rho^0$ last summer
- New result this winter for $\rho^+\rho^-$
  \[ \Rightarrow \text{updated CP} + f_L \text{ measurement} \]

\[
S_{\text{Long}} = -0.33 \pm 0.24^{+0.08}_{-0.14} \quad C_{\text{Long}} = -0.03 \pm 0.18 \pm 0.09 \\
 f_L = 0.978 \pm 0.014^{+0.021}_{-0.029} \]

- Updated Isospin analysis result

\[
\alpha = 100^\circ \pm 13^\circ
\]

- need to see $\rho^+\rho^0$ updated soon.