CKM Angles from $B \rightarrow VV$ Decays:

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OUTLINE

• We already have measurement of $\beta$ from $B_d \to J/\psi K_s$.

• We want to measure the other angles $\gamma$ and $\alpha$ and in many processes to test the SM. This talk is about measuring the CKM angles from $B \to VV$ decays.

• I will concentrate on
  a) $B \to D^* \bar{D}^*$, $B \to K^* \bar{K}^*$: These decays have $b \to d$ penguins.
  b) CKM angle using the full time dependent angular analysis including the interference terms: e.g. $B \to D^* \rho$

• The talk is based on ideas presented in
  J. Albert, A. Datta and D. London (HEP-PH 0410015)
  A. Datta and D. London (HEP-PH 0310252)
  D. London, N. Sinha and R. Sinha (HEP-PH 0005248)
There are 3 partial waves (helicity) amplitudes in $B \rightarrow V_1 V_2$.

One decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons are either longitudinal ($A_0$), or transverse to their directions of motion and parallel ($A_{||}$) or perpendicular ($A_{\perp}$) to one another.

\[ M = A_0 \varepsilon_1^L \cdot \varepsilon_2^L - \frac{1}{\sqrt{2}} A_{||} \bar{\varepsilon}_1^T \cdot \bar{\varepsilon}_2^T - \frac{i}{\sqrt{2}} A_{\perp} \bar{\varepsilon}_1^T \times \bar{\varepsilon}_2^T \cdot \hat{p} = A_0 g_0 + A_{||} g_{||} + i A_{\perp} g_{\perp} \]

where $\hat{p}$ is the unit vector along the direction of motion of $V_2$ in the rest frame of $V_1$, $\varepsilon_i^L = \bar{\varepsilon}_i \cdot \hat{p}$, and $\bar{\varepsilon}_i^T = \bar{\varepsilon}_i - \varepsilon_i^L \hat{p}$.

We will assume the different helicity amplitudes have been separated by an angular analysis. The polarization index will be dropped.
$B \rightarrow D^* \bar{D}^*$

$A^D = (T + E + P_c) V_{cb}^* V_{cd} + P_u V_{ub}^* V_{ud} + (P_t + P_{EW}^C) V_{tb}^* V_{td} = (T + E + P_c - P_t - P_{EW}^C) V_{cb}^* V_{cd} + (P_u - P_t - P_{EW}^C) V_{ub}^* V_{ud} \equiv A_{ct} e^{i\delta_{ct}} + A_{ut} e^{i\gamma} e^{i\delta_{ut}}$.
General Analysis

The general amplitude for $B^0 \rightarrow V_1V_2$ with a $b \rightarrow d$ penguin can be written for each polarization amplitude

$$A = A_{ut} e^{i\gamma} e^{i\delta_{ut}} + A_{ct} e^{i\delta_{ct}}$$

The time-dependent measurement of $B^0(t) \rightarrow V_1V_2$ allows one to obtain the three observables

$$B \equiv \frac{1}{2} \left( |A|^2 + |A|^2 \right) = A_{ct}^2 + A_{ut}^2$$
$$+ 2A_{ct} A_{ut} \cos \delta \cos \gamma$$

$$a_{dir} \equiv \frac{1}{2} \left( |A|^2 - |A|^2 \right) = -2A_{ct} A_{ut} \sin \delta \sin \gamma$$

$$a_{indir} \equiv \text{Im} \left( e^{-2i\beta} A^* \overline{A} \right)$$
$$- A_{ct}^2 \sin 2\beta - 2A_{ct} A_{ut} \cos \delta \sin(2\beta + \gamma)$$
$$- A_{ut}^2 \sin(2\beta + 2\gamma)$$

where $\delta \equiv \delta_{ut} - \delta_{ct}$
General Analysis

The three independent observables depend on five theoretical parameters: $A_{ut}, A_{ct}, \delta, \beta, \gamma$. Therefore one cannot obtain CP phase information from these measurements.

- Hence theoretical input is necessary to get the CKM phase information.

However, one can partially solve the equations to obtain

$$A^2_{ct} = \frac{a_R \cos(2\beta + 2\gamma) - a_{indir} \sin(2\beta + 2\gamma) - B}{\cos 2\gamma - 1} = f(\gamma, Exp)$$

$$a^2_R = B^2 - a^2_{dir} - a^2_{indir}$$

Hence if $A_{ct}$ is known then we can find $\gamma$

Get $A_{ct}$ from a partner process related by SU(3)
Flavour SU(3)

The SU(3) partner of $B_d \rightarrow D^* \bar{D}^*$ is $B_s \rightarrow D_{s}^* \bar{D}_s^*$ (Fleischer) and information from the $B_s$ decay can be used as the theoretical input.

However if exchange terms are neglected then we can use as the partner process $B_d \rightarrow D^* \bar{D}_s^*$.

Advantages: No $B_s$ required and SU(3) breaking is smaller.

Exchange terms are expected to be small from various theoretical reasons ($\sim 5\%$). More importantly the decays $B_d^0 \rightarrow D_s^{(+-)} \bar{D}_s^{(-*)}$ and $B_d^0 \rightarrow D_s^{(0*)} \bar{D}_s^{(0*)}$ proceed via exchange type diagrams.

Hence, a measurement of these rates relative to that of $B_d^0 \rightarrow D_s^{(+-)} \bar{D}_s^{(-*)}$ will provide an estimate of how important the $E$ contributions are.
$A_{ct}$ information can be obtained by considering the decay $B^{0}_d \rightarrow D_{s}^{*+} D^{-}$. This decay receives tree, $b \rightarrow s$ penguin and color-suppressed electroweak penguin contributions.
\[ A_{D_s}^* = (T' + P_c' - P_t' - P_{EW}'^C) V_{cb}^* V_{cs} + (P_u' - P_t' - P_{EW}'^C) V_{ub}^* V_{us} \]
\[ \approx (T' + P_c' - P_t' - P_{EW}'^C) V_{cb}^* V_{cs} \equiv A_{ct}' e^{i\delta_{ct}}. \]

The last line arises from the fact that
\[ |V_{ub}^* V_{us}/V_{cb}^* V_{cs}| \approx 2\%. \]

Thus, the measurement of the total rate for \( B_d^0 \to D_s^{++} D^{--} \) yields \( A_{ct}' \).

We now make the SU(3) assumption that

\[ \Delta \equiv \frac{\sin \theta_c A_{ct}'}{A_{ct}} = \frac{\sin \theta_c |(T' + P_c' - P_t' - P_{EW}'^C) V_{cb}^* V_{cs}|}{|(T + P_c - P_t - P_{EW}^C) V_{cb}^* V_{cd}|} = 1 \]

Given that \( A_{ct}' \) is measured in \( B_d^0 \to D_s^{++} D^{--} \), \( A_{ct} \) can be obtained from the above relation. This allows us to obtain \( \gamma \) from
\[ A_{ct}^2 = f(\gamma, Exp) \]
SU(3) breaking

\[ \Delta \equiv \frac{\sin \theta_c A'_{ct}}{A_{ct}} = \frac{\sin \theta_c |(T' + P'_c - P'_t - P'_{EW})V^*_{cb}V_{cs}|}{|(T + P_c - P_t - P^C_{EW})V^*_{cb}V_{cd}|} = 1 \]

Because \( T \) is the largest amplitude the leading SU(3) breaking comes from it.

The corrections from penguin amplitudes to \( \Delta = 1 \) are at the level of \( |P/T|(m_s/\Lambda^\chi) \sim 5\% \).

\[ \Delta \sim \frac{\sin \theta_c |T'V^*_{cb}V_{cs}|}{|TV^*_{cb}V_{cd}|} = \frac{F_0(B \rightarrow D^*)}{F_0(B \rightarrow D^*)} \frac{f_{D^*_s}}{f_{D^*}} + (m_s/\Lambda^\chi)(1/N_c^2) , \]

Note that if we had considered \( B_s \) decays there is additional SU(3) breaking from the form factors: \( \frac{F_0(B_s \rightarrow D^*_s)}{F_0(B \rightarrow D^*)} \).

So SU(3) breaking is from \( \frac{f_{D^*_s}}{f_{D^*}} \) which can be measured or calculated on lattice.
Reducing SU(3) breaking

It is possible to eliminate completely the dependence on decay constants by considering ratios of different polarization states.

\[
\frac{(A_{ct}^{\lambda_1})^2}{(A_{ct}^{\lambda_2})^2} = \frac{f^{\lambda_1}(\gamma)}{f^{\lambda_2}(\gamma)} = F(\gamma).
\]

The theoretical input is now provided by the assumption that

\[
\Delta' \equiv \frac{\Delta'_{\lambda'}}{\Delta_{\lambda}} = 1
\]

\[
\Delta_{\lambda'} \equiv \frac{\sin \theta_c A_{ct}^{'\lambda'}}{A_{ct}^{\lambda'}} = \sin \theta_c \frac{f_{D^*}}{f_{D^*}}
\]

\[
\Delta_{\lambda} \equiv \frac{\sin \theta_c A_{ct}^{\lambda'}}{A_{ct}^{\lambda}} = \sin \theta_c \frac{f_{D^*}}{f_{D^*}}
\]

The leading order SU(3) breaking \(\frac{f_{D^*}}{f_{D^*}}\) cancels in \(\Delta'\).
Net Theory Error

Second order SU(3) breaking from the tree amplitude: \((m_s/\Lambda_\chi)(1/N_c^2)\)

SU(3) breaking from Penguin: \(|P/T|(m_s/\Lambda_\chi) \sim 5\%\).

Factorizable SU(3) breaking in penguin also cancels in the double ratio \(\Delta'\).

Neglect of exchange terms \(\sim 5\%\).

Expect net theory error \(\sim 10\%\).

The relevant point is how does the theory error affect \(\gamma\): Depends on the Experimental numbers and the theory error appears to have no significant effect on \(\gamma\).
Present Status: $B \rightarrow D^* \bar{D}^*$

In order to extract $\gamma$, we have to separate the polarization amplitudes

The amplitudes $0$ and $||$ are CP-even, while $\perp$ is CP-odd. The data shows that the $D^* \bar{D}^*$ final state is almost entirely CP-even, i.e. the $\perp$ amplitude is negligible.

Unfortunately, at present experiments cannot distinguish between the $0$ and $||$ amplitudes. Have to work with combined $0$ and $||$ amplitudes

To combine $0$ and $||$ amplitudes we have to assume helicity independent non factorizable corrections: Additional assumption
The method presented is quite general and can be used to other decays, e.g. $B_{d,s} \rightarrow K^* \bar{K}^*$

The amplitude structure has the same form as $B \rightarrow D^* \bar{D}^*$

$$A(B^0_d \rightarrow K^* \bar{K}^*) = P_u V_{ub}^* V_{ud} + P_c V_{cb}^* V_{cd} + P_t V_{tb}^* V_{td}$$

$$= (P_c - P_t - P_{EW}^C) V_{cb}^* V_{cd} + (P_u - P_t - P_{EW}^C) V_{ub}^* V_{ud}$$

$$\equiv \mathcal{A}_{ct} e^{i\delta_{ct}} + \mathcal{A}_{ut} e^{i\gamma} e^{i\delta_{ut}}$$

CKM Angles from $B \rightarrow VV$ Decays: – p.14
CKM angle from Full Angular Analysis

Consider a final state $f$, consisting of two vector mesons, to which both $B^0$ and $\bar{B}^0$ can decay. We assume further that only one weak amplitude contributes to $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$. We write the helicity amplitudes as follows:

$$A_{\lambda} \equiv Amp(B^0 \rightarrow f)_{\lambda} = a_{\lambda} e^{i \delta_{\lambda}^{\alpha}} e^{i \phi_{\alpha}} ,$$  \hspace{1cm} (1) $$A'_{\lambda} \equiv Amp(\bar{B}^0 \rightarrow f)_{\lambda} = b_{\lambda} e^{i \delta_{\lambda}^{b}} e^{i \phi_{b}} ,$$  \hspace{1cm} (2) $$\bar{A}'_{\lambda} \equiv Amp(B^0 \rightarrow \bar{f})_{\lambda} = b_{\lambda} e^{i \delta_{\lambda}^{b}} e^{-i \phi_{b}} ,$$  \hspace{1cm} (3) $$\bar{A}_{\lambda} \equiv Amp(\bar{B}^0 \rightarrow \bar{f})_{\lambda} = a_{\lambda} e^{i \delta_{\lambda}^{\alpha}} e^{-i \phi_{\alpha}} ,$$  \hspace{1cm} (4)

The time-dependent decay rate for $B^0(t) \rightarrow f$ can be written as

$$\Gamma(B^0(t) \rightarrow f) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left( \Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma} \cos(\Delta M t) \right) - \rho_{\lambda\sigma} \sin(\Delta M t) g_{\lambda} g_{\sigma} .$$
Similarly, the decay rate for \( B^0(t) \rightarrow \bar{f} \) is given by

\[
\Gamma(B^0(t) \rightarrow \bar{f}) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left( \bar{A}_{\lambda \sigma} + \bar{\Sigma}_{\lambda \sigma} \cos(\Delta M t) - \bar{\rho}_{\lambda \sigma} \sin(\Delta M t) \right) g_{\lambda} g_{\sigma}.
\]

where the \( g_\lambda \) are the coefficients of the helicity amplitudes written in the linear polarization basis. The \( g_\lambda \) depend only on the angles describing the kinematics.
The Diagonal Terms

The diagonal terms are

\[ \Lambda_{\lambda\lambda} = \Lambda_{\lambda\lambda} = \frac{(a_\lambda^2 + b_\lambda^2)}{2}, \Sigma_{\lambda\lambda} = -\bar{\Sigma}_{\lambda\lambda} = \frac{(a_\lambda^2 - b_\lambda^2)}{2}. \]

\[ \rho_{\lambda\lambda} = \pm a_\lambda b_\lambda \sin(\phi + \delta_\lambda), \bar{\rho}_{\lambda\lambda} = \pm a_\lambda b_\lambda \sin(\phi - \delta_\lambda), \]

\[ \phi \equiv -2\phi_M + \phi_b - \phi_a \] and \( \delta_\lambda \) is the relative phase between \( a_\lambda \) and \( b_\lambda \)

So far we have worked with the diagonal terms but now we get information from the interference terms also
The Interference terms

The interference terms can be expressed in terms of amplitudes- very messy!

The Triple Products are CP violating observables involving the T-odd triple product $\tilde{p}.(\epsilon_1 \times \epsilon_2)$

$$T.P = \Lambda_{\perp i} + \tilde{\Lambda}_{\perp i}$$

For a decay dominated by a single amplitude $T.P=0$ and a good place to look for new physics specially in modes like $B \to \phi K^*, B \to \rho K^*, B_{d,s} \to K^* \bar{K}^*$ etc (Datta and London)

The other interference terms $\Sigma_{\perp i} = \tilde{\Sigma}_{\perp i}, \rho_{\perp i}$ and $\tilde{\rho}_{\perp i}$ can be manipulated to get the $\sin^2 \phi (\phi = -2\phi_M + \phi_b - \phi_a)$
Here $\sin^2 \phi = \sin^2 (2\beta + \gamma)$ can be extracted.

Potential problem:
$Amp(\bar{B}_d^0 \rightarrow D^{**} \rho^-) / Amp(\bar{B}_d^0 \rightarrow D^{**} \rho^-) \sim |V_{ub}V_{cd}^* / V_{cb}^* V_{ud}| \sim 2 \times 10^{-2}$. This results in a very small CP-violating asymmetry.

However the measurement does not depend on the square of the small amplitude (only linearly) as in the PP case (Dunietz).
Conclusions

I have presented methods to get CKM angles from $B \rightarrow VV$ Decays

The first method relies on the diagonal terms in the full time dependent angular analysis and uses SU(3). Can be applied to many pairs of decays ($B \rightarrow D^*D^*$ and $B \rightarrow D_s^*D^*$)

The second method relies on the full time dependent angular analysis including interference terms. Works for decays with a single amplitude e.g. $B(\bar{B}) \rightarrow D^*\rho$. 