Charm input for $\gamma$ measurements (theory)

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1. Introduction: why do we care about charm?

- **Standard Model is a very constrained system**
  - a single CP-violating parameter

- **Dual role of charm physics:**
  - supporting measurements for B-physics CKM methods
  - direct CKM information from charm transitions ($V_{cs}$, $V_{cd}$)
  - possible New Physics effects in charm
Extraction of the CKM angle $\gamma$:

Cleanest signals involve interference of $b \to \overline{c}u\overline{s}$ and $b \to u\overline{c}s$.

- **via** $B^\pm \to D\left(D^0, D^0, D_{CP}\right) \to f)K^\pm$ (Gronau, Wyler, London)
- **via** $B^\pm \to D\left(D^0, D^0\right) \to K\pi K^\pm$ (Atwood, Dunietz, Soni)
- **via** $B^\pm \to D\left(D \to KK^*\right)K^\pm$ (Grossman, Ligeti, Soffer)
- **via** $B^\pm \to D\left(D \to \text{multibody}\right)K^\pm$ (Giri, Grossman, Soffer, Zupan, Atwood, Soni)


All of those methods will benefit from charm measurements.
2a. GWL methods and charm inputs

Use CP-eigenstate decay channels of the D ($D^0/\overline{D}^0 \rightarrow f_{CP}$)...

\[
R_{CP^\pm} = \frac{\Gamma \left[ B^- \rightarrow D_{CP^\pm}^0 K^- \right] + \Gamma \left[ B^+ \rightarrow D_{CP^\pm}^0 K^+ \right]}{\Gamma \left[ B^- \rightarrow D^0 K^- \right]} = 1 \pm 2r_B \cos \gamma \cos \delta_B + r_B^2
\]

\[
A_{CP^\pm} = \frac{\Gamma \left[ B^- \rightarrow D_{CP^\pm}^0 K^- \right] - \Gamma \left[ B^+ \rightarrow D_{CP^\pm}^0 K^+ \right]}{\Gamma \left[ B^- \rightarrow D_{CP^\pm}^0 K^- \right] + \Gamma \left[ B^+ \rightarrow D_{CP^\pm}^0 K^+ \right]} = \pm 2r_B \sin \gamma \sin \delta_B / R_{CP^\pm}
\]

...if we define

\[
\frac{A \left( B^+ \rightarrow D^0 K^+ \right)}{A \left( B^+ \rightarrow \overline{D}^0 K^+ \right)} = r_B e^{i\gamma} e^{i\delta_B}
\]

D-$\overline{D}$ mixing might affect determination of $\gamma$...
Charm mixing?

\[
\begin{align*}
\Gamma \left[ B^+ \to f_{CP}K^+ \right] & \propto 1 + r_B^2 + 2r_B \cos \left( \gamma + \delta_B \right) \\
\Gamma \left[ B^- \to f_{CP}K^- \right] & \propto 1 + r_B^2 + 2r_B \cos \left( \gamma - \delta_B \right)
\end{align*}
\]

\[\begin{align*}
\Gamma & \propto \left(1 - c_+ c_- \right) \left(1 - c_+^2 \right) \left(1 - c_-^2 \right)
\end{align*}\]

- negligible D-mixing:

\[
\sin^2 \gamma = \frac{1}{2} \left[ 1 - c_+ c_- \pm \sqrt{(1 - c_+^2)(1 - c_-^2)} \right]
\]

- non-negligible D-mixing:

\[
c_+ = \cos(\gamma + \delta_B) - \frac{x_D}{2r_B} \sin(2\theta_D) - \frac{y_D}{2r_B} \left[ 2\eta_f r_B \cos(\gamma + 2\theta_D + \delta_B) + \cos(2\theta_D) \right]
\]

\[
c_- = \cos(\gamma - \delta_B) + \frac{x_D}{2r_B} \sin(2\theta_D) - \frac{y_D}{2r_B} \left[ 2\eta_f r_B \cos(\gamma + 2\theta_D - \delta_B) + \cos(2\theta_D) \right]
\]

Constraining DD mixing helps in determination of \(\gamma\)

Sizable effect if \(y_D \sim 1\%\)

J.P. Silva and A. Soffer

Alexey Petrov (WSU)

CKM 2005, March 15-19, UCSD
Charm mixing: theoretical estimates

Updated predictions
A.A.P. hep-ph/0311371

Theoretical predictions are all over the board... so:

- Are $x, y \sim 1\%$ theoretically possible?

- What is the relationship between $x$ and $y$ ($x \sim y$, $x > y$, $x < y$) in the Standard Model?

- What are the experimental constraints?

(papers from SPIRES)

- $x$ from new physics
- $y$ from Standard Model
- $\Delta$ from Standard Model

Falk, Grossman, Ligeti, Nir, and A.A.P.
Phys. Rev. D65, 054034, 2002
Phys. Rev. D69, 114021, 2004

Updated predictions

| $|x|$ or $|y|$ |
|----------------|
| 1   3   5   7   9 11 13 15 17 19 21 23 25 27 29 31 |

- theoretical predictions (from SPIRES)
Experimental constraints on charm mixing

Idea: look for a wrong-sign final state

1. Time-dependent or time-integrated semileptonic analysis

\[ \text{rate} \propto x^2 + y^2 \]

Quadratic in x,y: not so sensitive

2. Time-dependent $D^0(t) \rightarrow K^+K^-$ analysis (lifetime difference)

\[ y_{CP} = \frac{\tau(D \rightarrow \pi^+K^-)}{\tau(D \rightarrow K^+K^-)} - 1 = y \cos \phi - x \sin \phi \frac{1 - R_m}{2} \]

World average: $y_{CP} = (0.9 \pm 0.4)\%$

3. Time-dependent $D^0(t) \rightarrow K^+\pi^-$ analysis

\[
\Gamma[D^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} |A_{K^+\pi^-}|^2 \left[ R + \sqrt{R R_m} (y^' \cos \phi - x^' \sin \phi) \Gamma t + \frac{R_m^2}{4} (y^2 + x^2) (\Gamma t)^2 \right]
\]

Sensitive to DCS/CF strong phase $\delta$

\[ R_m^2 = \left| \frac{q}{p} \right|^2, \quad x^' = x \cos \delta + y \sin \delta, \quad y^' = y \cos \delta - x \sin \delta \]

Alexey Petrov (WSU)
Quantum coherence: new mixing studies

1. Time-integrated $D^0 \to K^+\pi^-$ analysis: DCSD contribution cancels out for double-tagged $D^0\bar{D}^0 \to (K^-\pi^+)(K^-\pi^+)$ decays!

Indeed, consider $L=I$: $\left| D^0\bar{D}^0 \right\rangle = \frac{1}{\sqrt{2}} \left[ \left| D^0(k_1)\bar{D}^0(k_2) \right\rangle - \left| D^0(k_2)\bar{D}^0(k_1) \right\rangle \right]$ so

$$ A\left(D^0\bar{D}^0 \to (K^-\pi^+)^2\right) = \frac{1}{\sqrt{2}} \left\langle (K^-\pi^+)(K^-\pi^+)|H_{\text{eff}}|D^0\bar{D}^0 \right\rangle - \frac{1}{\sqrt{2}} \left\langle (K^-\pi^+)(K^-\pi^+)|H_{\text{eff}}|\bar{D}^0D^0 \right\rangle $$

... which implies for $A$

$$ A\left(D^0\bar{D}^0 \to (K^-\pi^+)^2\right) \propto A_{\text{CF}}/A_{\text{DCS}} - A_{\text{DCS}}A_{\text{CF}} - \frac{1}{2} \frac{p}{q} \frac{\Delta t}{\Gamma} (y + ix) $$

and

$$ R\left(\frac{(K^-\pi^+)(K^-\pi^+)}{(K^-\pi^+)(K^+\pi^-)}\right) = \frac{x^2 + y^2}{2} \left| \frac{p}{q} \right|^2 = r_{D}^2 \left| \frac{p}{q} \right|^2 $$

With 3 fb$^{-1}$ of data $r$ can be determined to $r_{D} < 2\%$!

Yamamoto; Bigi, Sanda

Quadratic in $x,y$: not so sensitive wanted: linear in $x$ or $y$
2b. ADS methods and charm inputs

Use other common decay channels of the D ($D^0/D^0 \rightarrow K^+\pi^-$)

$$R_{ADS} = \frac{\Gamma\left[ B^- \rightarrow D^0 \left( \rightarrow K^+\pi^- \right) K^- \right]}{\Gamma\left[ B^- \rightarrow D^0 \left( \rightarrow K^-\pi^+ \right) K^- \right]} + \frac{\Gamma\left[ B^+ \rightarrow D^0 \left( \rightarrow K^-\pi^+ \right) K^+ \right]}{\Gamma\left[ B^+ \rightarrow D^0 \left( \rightarrow K^+\pi^- \right) K^+ \right]}$$

$$= r_D^2 + 2r_Br_D \cos \gamma \cos (\delta_B + \delta_D) + r_B^2$$

$$A_{ADS} = \frac{\Gamma\left[ B^- \rightarrow D^0 \left( \rightarrow K^+\pi^- \right) K^- \right]}{\Gamma\left[ B^- \rightarrow D^0 \left( \rightarrow K^-\pi^+ \right) K^- \right]} - \frac{\Gamma\left[ B^+ \rightarrow D^0 \left( \rightarrow K^-\pi^+ \right) K^+ \right]}{\Gamma\left[ B^+ \rightarrow D^0 \left( \rightarrow K^+\pi^- \right) K^+ \right]}$$

$$= 2r_B r_D \sin \gamma \sin (\delta_B + \delta_D) / R_{ADS}$$

...if we define

$$\frac{A\left( D^0 \rightarrow K^+\pi^- \right)}{A\left( D^0 \rightarrow K^-\pi^+ \right)} = r_D e^{i\delta_D}$$

Determine at charm factory

Measurement of charm parameters will help determination of $\gamma$

What should we expect?
Theoretical expectation: PP final states

Strong phase $\delta_D$ is zero in flavor SU(3) limit (only U-spin is needed):

$$A_{CF} \left( D^0 \rightarrow K^+ \pi^- \right)$$

$$A_{DCS} \left( D^0 \rightarrow K^- \pi^+ \right)$$

U-spin

... but flavor SU(3) is badly broken in D-decays...

Phase $\delta_D$ is strongly model-dependent!

$$A = A_T + A_R e^{i\phi}$$

$$B = B_T + B_R e^{i\phi}$$

$$A_R = \frac{g_H}{m_D^2 - m_{K_H}^2 + i\Gamma_{K_H} m_D} \langle K_H | H_{eff} | D^0 \rangle,$$

$$B_R = \frac{g_H}{m_D^2 - m_{K_H}^2 + i\Gamma_{K_H} m_D} \langle K_H | H_{eff}^\dagger | D^0 \rangle,$$

$$R_{exp} = \frac{A_{K^+\pi^-}}{A_{K^+\pi^-}}$$

A. Falk, Y. Nir and A.A.P., JHEP 12 (1999) 019
Quantum coherence: measuring strong phases

- If CP violation in charm is neglected: mass eigenstates = CP eigenstates

\[ |D_{CP}^\pm \rangle = \frac{1}{\sqrt{2}} \left[ |D^0 \rangle \pm |\overline{D}^0 \rangle \right] \]

- CP eigenstates do NOT evolve with time, so can be used for "tagging"

\[ |D_{CP}^+ \rangle \rightarrow f_1 f_2 \]

- CLEO-c has good CP-tagging capabilities
  CP anti-correlated \( \psi(3770) \): CP(tag) \((-1)^L = [CP(K_S) CP(\pi^0)] (-1) = +1 \)
  CP correlated \( \psi(4140) \)
Quantum coherence: measuring strong phases

Strong phase can be measured at CLEO-c/BES-III!

With a particular choice of phase conventions...

\[ \sqrt{2} A(D_{CP\pm} \to K^-\pi^+) = A(D^0 \to K^-\pi^+) \pm A(D^0 \to K^-\pi^+) \]

\[ \sqrt{2} A(D_{CP\pm} \to K^-\pi^+) \]

\[ A(D^0 \to K^-\pi^+) \]

In the limit of CP-invariance

\[ A(D^0 \to K^+\pi^-) = A(D^0 \to K^-\pi^+) \]

\[ \cos \delta_D = \frac{Br(D_{CP+} \to K^-\pi^+) - Br(D_{CP-} \to K^-\pi^+)}{2\sqrt{R} Br(D^0 \to K^-\pi^+)} \]

Other observables (same idea) are possible with measured \( A_{CP} \)

Silva, Soffer; Gronau, Grossman, Rosner

With 3 fb\(^{-1}\) of data \( \cos \delta \) can be determined to \( |\Delta \cos \delta| < 0.05! \)
2. Can also use flavor-specific (semileptonic) decays in combination with CP-tagging to study mixing

\[ R^L_\sigma = \frac{1}{B_r(D^0 \rightarrow X l \nu)} \frac{\Gamma[\psi_L \rightarrow (D \rightarrow [CP^\sigma](D \rightarrow X l \nu))]}{\Gamma[\psi_L \rightarrow (D \rightarrow [CP^\sigma](D \rightarrow X ))]} \]

... which implies for \( y \)

\[ y \cos \phi = (-1)^L \sigma \frac{R^L_\sigma - 1}{R^L_\sigma} \]

... or alternatively if \( B^l_\pm = \frac{\Gamma(D_{CP^\pm} \rightarrow X l \nu)}{\Gamma_{tot}} \)

\[ y \cos \phi = \frac{1}{4} \left( \frac{B^l_+}{B^l_-} - \frac{B^l_-}{B^l_+} \right) \]

Now, 3 fb\(^{-1}\) of data gives approximately 17.4 million \( D^0 D^0 \) pairs. CLEO-c can reconstruct 800K CP+ and 200K CP- final states.

This implies that for, say, \( y \sim 0.7\% \) one gets \( y = 0.7 \pm 0.4 \% \), i.e. sensitivity to \( y \) is down to a fraction of a percent!

D. Atwood, A.A.P., hep-ph/0207165
VP final states

Use VP common decay channels of the $D (D^0/D^0 \rightarrow K^{*+}\pi^-, \rho^- K^+, \rho^+ \pi^-)$

Strong phase can again be measured at CLEO-c/BES-III!

$$\sqrt{2} \, A \left( D_{CP\pm} \rightarrow K^{*-}\pi^+ \right) = A \left( D^0 \rightarrow K^{*-}\pi^+ \right) \pm A \left( D^0 \rightarrow K^{*-}\pi^+ \right)$$

In the limit of CP-invariance

$$A \left( D^0 \rightarrow K^{*-}\pi^- \right) = A \left( D^0 \rightarrow K^{*-}\pi^+ \right)$$

Thus again,

$$\cos \delta_D^* = \frac{Br \left( D_{CP+} \rightarrow K^{*-}\pi^+ \right) - Br \left( D_{CP-} \rightarrow K^{*-}\pi^+ \right)}{2\sqrt{R \, Br \left( D^0 \rightarrow K^{*-}\pi^+ \right)}}$$

This analysis is true for any non-CP eigenstate decay of $D$!

What should one expect for $\delta_D^*$??
Speculation: chiral symmetry in QCD is realized in the “vector” mode

1. Goldstone bosons, but no SSB \(\Rightarrow\) parity doubles
2. Massless scalars are longitudinal components of vector mesons

D\(\rightarrow\)VP decays involve only longitudinal components of V \(\Rightarrow\) Relate PP and VP amplitudes and phases!

Under SU(3)\(_L\)\(\times\)SU(3)\(_R\) D-meson transforms as \(D^0 \sim (\bar{3},1)+(1,\bar{3})\), so

\[
D^0_L = D^0 + D^0_S \sim (\bar{3},1), \quad D^0_R = D^0 - D^0_S \sim (1,\bar{3})
\]

Chiral partner of P (i.e. \(\pi,\eta,K\)) is longitudinal component of V (i.e. \(\rho,\omega,K^*\)), \(V_{\text{long}}\)

\[
P_L = P + V_{\text{long}} \sim (8,1), \quad P_R = P - V_{\text{long}} \sim (1,8)
\]

The \(\Delta C=1\) part of \(H_W\) is \(\left(\bar{c}q_i\right)\left(\bar{q}_j q_k\right)\), i.e. \(15 + \bar{6}\) and purely left-handed
VP strong phases: vector symmetry

Speculation: chiral symmetry in QCD is realized in the “vector” mode

Interested in \( \langle P_{L(R)} P_{L(R)} | H_W | D_{L(R)} \rangle \)

...but since \( \langle P_R P | H_W | D_{L(R)} \rangle = \langle P_R V_{long} | H_W | D \rangle = 0 \)

\[ \langle PP | H_W | D \rangle = \langle PV | H_W | D \rangle = \langle V_{long} V_{long} | H_W | D \rangle \]

Strong phase \( \delta_D^* \) is zero in the vector SU(3)_L x SU(3)_R limit:

\[ A_{CF} \left( D^0 \to K^* \pi^- \right) \]

\[ A_{DCS} \left( D^0 \to K^- \pi^+ \right) \]

... but vector symmetry is badly broken...

H. Georgi, F. Uchiyama

Alexey Petrov (WSU)
CKM 2005, March 15-19, UCSD
Nonresonant multibody final states

Use the whole Dalitz plot of the D decay \((D^0/\bar{D}^0 \to K_S\pi^+\pi^-)\)!

\[
A_D(s_{12}, s_{13}) = A(D^0 \to K_S(p_1)\pi^- (p_2)\pi^+(p_3)) = a_0 e^{i\delta_0} + \sum a_r e^{i\delta_r} A_r(s_{12}, s_{13})
\]

Plug into the amplitude...

\[
A(B^- \to D[\to K_S\pi^+\pi^-]K^-) = A_B P_D \left( A_D(s_{12}, s_{13}) + r_B e^{i(\delta_{b^-} - \gamma)} A_D(s_{13}, s_{12}) \right)
\]

...or integrate over portions of Dalitz plot

\[
c_i \equiv \int dp \ A_{12,13} A_{13,12} \cos(\delta_{12,13} - \delta_{13,12}),
\]

\[
s_i \equiv \int dp \ A_{12,13} A_{13,12} \sin(\delta_{12,13} - \delta_{13,12}),
\]

\[
T_i \equiv \int dp \ A_{12,13}^2
\]
Conclusions

• Charm physics plays a dual role in CPV effort:
  - Supporting measurements for B-physics
  - Direct searches for New Physics

• Important measurements for clean extraction of $\gamma$
  - GLW methods (mixing)
  - ADS methods (strong phases)
  - There is a symmetry limit where PP and PV phases are exactly zero

• Expect new results!
Additional slides
b. Unitarity relation cross-checks

- Several triangle unitarity relations involving charm inputs

\[ V_{td} V_{cd}^* + V_{ts} V_{cs}^* + V_{tb} V_{cb}^* = 0 \]

\[ \lambda^4 \quad \lambda^2 \quad \lambda^2 \]

\[ V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0 \]

\[ \lambda \quad \lambda \quad \lambda^5 \]

\[ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \]

\[ \lambda \quad \lambda \quad \lambda^5 \]

All of them have the same area > useful cross-checks

Bigi and Sanda

“Tree-level” triangle (no NP)

Alexey Petrov (WSU)
New Physics: role of charm

Same program can be realized with charm transitions

1. Processes forbidden in the Standard Model to all orders (or very rare)
   
   Examples: \( D^0 \rightarrow \mu^+ \mu^+ \), \( D^0 \rightarrow \mu^+ e^- \)

2. Processes forbidden in the Standard Model at tree level
   
   Examples: \( D^0 - D^0 \) mixing, \( D \rightarrow X\gamma \), \( D \rightarrow X\nu\bar{\nu} \)

3. Processes allowed in the Standard Model
   
   Examples: relations, valid in the SM, but not necessarily in general

Start from the bottom…
Charm mixing: basics

\[ \Delta Q = 2: \text{only at one loop in the Standard Model: possible new physics particles in the loop} \]

\[ \Delta Q = 2 \text{ interaction couples dynamics of } B^0 \text{ and } \bar{B}^0 \]

or \( D^0 \text{ and } \bar{D}^0 \)

\[ |D(t)\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = a(t) |D^0\rangle + b(t) |\bar{D}^0\rangle \]

• Time-dependence: coupled Schrödinger equations

\[ i \frac{\partial}{\partial t} |D(t)\rangle = \left( M - \frac{i}{2} \Gamma \right) |D(t)\rangle = \begin{bmatrix} A \\ p^2 q^2 \end{bmatrix} \begin{bmatrix} A \\ p^2 A \end{bmatrix} |D(t)\rangle \]

• Diagonalize: mass eigenstates ≠ flavor eigenstates

\[ |D_{1,2}\rangle = p |D^0\rangle \pm q |\bar{D}^0\rangle \]

Mass and lifetime differences of mass eigenstates:

\[ x = \frac{M_2 - M_1}{\Gamma}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2 \Gamma} \]
Why do we care?

- intermediate up-type quarks
- SM: $t$-quark contribution is dominant (expected to be large)

- intermediate down-type quarks
- SM: $b$-quark contribution is negligible due to $V_{cd}V_{ub}^*$ (zero in the SU(3) limit)

$D^0 - D^0$ mixing

\[ D^0 - D^0 \text{ mixing} \]

1. Sensitive to long distance QCD
2. Small in the SM: New Physics!
   (must know SM $x$ and $y$)

$B^0 - B^0$ mixing

\[ B^0 - B^0 \text{ mixing} \]

1. Computable in QCD (*)
2. Large in the SM: CKM!

(*) up to matrix elements of 4-quark operators


2nd order effect!!!
How would new physics affect mixing?

- Look again at time development:

\[ i \frac{\partial}{\partial t} |D(t)\rangle = \left(M - \frac{i}{2} \Gamma \right) |D(t)\rangle = \begin{bmatrix} A & p^2 \\ q^2 & A \end{bmatrix} |D(t)\rangle \]

- Expand \( D^0 - D^0 \) mass matrix:

\[
\left(M - \frac{i}{2} \Gamma \right)_{ij} = m_D^{(0)} \delta_{ij} + \frac{1}{2m_D} \langle D^0_i | H_{w}^{AC=2} | D^0_j \rangle + \frac{1}{2m_D} \sum_l \frac{\langle D^0_i | H_{w}^{AC=1} | I \rangle \langle I | H_{w}^{AC=1} | D^0_j \rangle}{m_D^2 - m_l^2 + i\varepsilon} \]

Local operator, affects \( x \), possible new physics

Real intermediate states, affect both \( x \) and \( y \) ⇒ Standard Model

1. \( x >> y \) : signal for New Physics?
2. \( x \approx y \) : Standard Model?

2. CP violation in mixing/decay

\{ With b-quark contribution neglected: \
- only 2 generations contribute \\ ⇒ real 2x2 Cabibbo matrix \}
Quantum coherence: supporting measurements

Time-dependent $D^0(t) \to K^+\pi^-$ analysis

$$\Gamma[D^0(t) \to K^+\pi^-] = e^{-\Gamma t} |A_{K^+\pi^-}|^2 \left[ R + \sqrt{R} R_m (y' \cos \phi - x' \sin \phi) \Gamma t + \frac{R_m^2}{4} (y'^2 + x'^2)(\Gamma t)^2 \right]$$

where $R = \left| \frac{A_{K^+\pi^-}}{A_{K^+\pi^-}} \right|^2$ and $x' = x \cos \delta + y \sin \delta$

$y' = y \cos \delta - x \sin \delta$

Strong phase $\delta$ is zero in the SU(3) limit and strongly model-dependent

A. Falk, Y. Nir and A.A.P., JHEP 12 (1999) 019

Strong phase can be measured at CLEO-c!

$$\sqrt{2} A(D_{CP\pm} \to K^-\pi^+) = A(D^0 \to K^-\pi^+) \pm A(D^0 \to K^-\pi^+)$$

$$\cos \delta = \frac{Br(D_{CP+} \to K^-\pi^+) - Br(D_{CP-} \to K^-\pi^+)}{2\sqrt{R} Br(D^0 \to K^-\pi^+)}$$

With 3 fb$^{-1}$ of data $\cos \delta$ can be determined to $|\Delta \cos \delta| < 0.05!$