Lifetime Ratios: \( \frac{\tau(B^+)}{\tau(B_d)}, \frac{\tau(B_s)}{\tau(B_d)}, \frac{\tau(\Lambda_b)}{\tau(B_d)} \)

Width Differences: \( \Delta\Gamma_{B_d}, \Delta\Gamma_{B_s} \)

**Beauty Hadrons:**

Cecilia Tarantino
Università Roma Tre and INFN Sezione di Roma III
\[ B^0_q - \overline{B}^0_q \]

**SYSTEMS** (q=d,s)

\[ \hat{H} = \hat{M} \frac{i}{2} \hat{\Gamma} \]

\[ \hat{M} = \begin{pmatrix} M_{11} & M_{21}^* \\ M_{21} & M_{11} \end{pmatrix} \]

\[ \hat{\Gamma} = \begin{pmatrix} \Gamma_{11} & \Gamma_{21}^* \\ \Gamma_{21} & \Gamma_{11} \end{pmatrix} \]

**Oscillation**

**Eigenstates:**

\[ \left| B^{L,H}_q \right> = p_q \left| B^0_q \right> \pm q_q \left| \overline{B}^0_q \right> \]

\[ \Gamma^q_{11} \quad (\Delta B = 0) \]

**Lifetimes:** \[ \Gamma_{B_q} = \Gamma^q_{11} \]

\[ \Gamma^q_{21} \quad (\Delta B = 2) \]

**Width Differences:** \[ \Delta \Gamma_{B_q} = \Gamma^q_{B^L_q} - \Gamma^q_{B^H_q} \leftrightarrow \Gamma^q_{21} \]
\[ \Gamma_{11}^q \propto \text{Disc}\langle B_0^q | T | B_0^q \rangle \quad \Gamma_{21}^q \propto \text{Disc}\langle \overline{B}_0^q | T | B_0^q \rangle \]

\[ T = i \int d^4x \ T (\mathcal{H}_{\text{eff}}^{\Delta B=1}(x) \mathcal{H}_{\text{eff}}^{\Delta B=1}(0)) \]

\[ \text{HQE} \]

\[ (m_b >> \Lambda_{QCD}) \]

Large energy release \hspace{1cm} \leftrightarrow \hspace{1cm} contact interaction

SPECTATOR EFFECTS

\[ \Gamma_{21}^q = \sum_k \frac{\mathcal{C}_k^q(\mu)}{m_b^3} \langle B_0^0 | \overline{O}_k^{\Delta B=2}(\mu) | B_0^0 \rangle \]

\[ \mathcal{C}_k^q(\mu) \]: short-distance (perturbative)

\[ \langle B_0^0 | \overline{O}_k^{\Delta B=2}(\mu) | B_0^0 \rangle \]: long-distance (non-perturbative)
Wilson coefficients

\[ \text{Disc } \langle T \rangle = \bar{c}(\mu) \cdot \langle \tilde{O}(\mu) \rangle \]

Matching at the NLO in QCD and \( O(1/m_b) \) in the HQE

\[ \Delta B = 2(\Delta B = 0) \]

NLO Diagrams
**Lifetime Ratios**

\[
\Gamma = \frac{G_F^2 V_{cb}^2 m_b^5}{192\pi^3 (2M_B)} \left[ c^{(3)} \langle \bar{b} b \rangle + c^{(5)} \frac{g_s}{m_b^2} \langle \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b \rangle + \frac{96\pi^2}{m_b^3} \sum_k \left( c^{(6)}_k \langle O^{(6)}_k \rangle + c^{(7)}_k \langle O^{(7)}_k \rangle \right) \right]
\]

\( O(1) \) (1996) [M. Neubert and C.T. Sachrajda]

\( O(\alpha_s) \) (2002)

[E. Franco, V. Lubicz, F. Mescia and C.T.]


\( O(1/m_b) \) (2004)

[F. Gabbiani, A. I. Onishchenko and A. A. Petrov]

**Width Differences**

\( O(\alpha_s) \) (2003)

[M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C.T.]


\( O(1/m_b) \) (1996)


\( O(1/m_b^2) \) (in progress)

[A. Lenz and U. Nierste]
Matrix Elements:  \( \Delta B = 0 \)  

Operators  

Leading spectator effect contribution  

\[
O_1^g = (\bar{b}q)_{V-A} (\bar{q}b)_{V-A} \leftrightarrow B_1^g, \\
O_2^g = (\bar{b}q)_{S-P} (\bar{q}b)_{S+P} \leftrightarrow B_2^g, \\
O_3^g = (\bar{b}T^a q)_{V-A} (\bar{q}T^a b)_{V-A} \leftrightarrow \epsilon_1^g, \\
O_4^g = (\bar{b}T^a q)_{S-P} (\bar{q}T^a b)_{S+P} \leftrightarrow \epsilon_2^g, 
\]

\[
O_P = (\bar{b}T^a b)_V \sum_{q=u,d,s,c} (\bar{q}T^a q)_V
\]

\[
[ (\bar{q}q)_{V-A} = \bar{q}\gamma_5^\mu q, \quad (\bar{q}q)_{S\pm P} = \bar{q}(1 \pm \gamma_5)q, \quad (\bar{q}q)_V = \bar{q}\gamma^\mu q ]
\]

B-parameters:  

- \( \Lambda_b \): computed in lattice-HQET, evolved at the LO  

\[ B_d - B_s - B^+ \] **B-parameters**

- **Lattice-HQET**
  \[
  B_1^d = 1.06 \pm 0.08, \quad B_2^d = 1.01 \pm 0.07, \\
  \epsilon_1^d = -0.01 \pm 0.03, \quad \epsilon_2^d = -0.03 \pm 0.02.
  \]

- **Lattice-QCD**
  \[
  B_1^d = 1.2 \pm 0.2, \quad B_2^d = 0.9 \pm 0.1, \\
  \epsilon_1^d = 0.04 \pm 0.01, \quad \epsilon_2^d = 0.04 \pm 0.01.
  \]
  [APE (D. Becirevic et al.), 2001]

- **Sum Rules, in HQET**
  \[
  B_1^d = 1.01 \pm 0.01, \quad B_2^d = 0.99 \pm 0.01, \\
  \epsilon_1^d = -0.08 \pm 0.02, \quad \epsilon_2^d = -0.01 \pm 0.03.
  \]
  [M.S. Baek et al., 1998]

**Subleading spectator effect contribution**

\[ O(1/m_b^4) \]

8 operators, from the VSA (B-mesons) or the quark-diquark model (baryon)
Matrix Elements: \( \Delta B = 2 \) Operators

**Leading contribution** \( O(1/m_b^3) \)

\[
\mathcal{O}_1^q = (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} \leftrightarrow B_1^q, \quad \mathcal{O}_2^q = (\bar{b}q)_{S-P} (\bar{b}q)_{S-P} \leftrightarrow B_2^q.
\]

From the *lattice* (with different methods) or QCD *sum rules* [J.G. Korner et al., 2003]

**Subleading contribution** \( O(1/m_b^4) \)

4 operators \( (R_1^q, R_2^q, R_3^q, R_4^q) \):

- \( R_1^q, R_4^q \): related, through Fierz and eq. of motion, to operators computed on the *lattice*
- \( R_2^q, R_3^q \): from the *VSA*
• **HQET** \((m_b \to \infty)\)
  
  \[ B^1_q, B^2_q \]

  \[ B^{s}_1 = 0.83(5)(6), \quad B^{s}_2 = 0.81(2)(10) \]

  [V. Gimenez and J. Reyes, 2000]

• **NRQCD** \(O(1/m_b)\)
  
  \[ B^{s}_1 = 0.85(3)(11), \quad B^{s}_2 = 0.82(2)(11) \]

  [Hi-KEK (S. Hashimoto et al.), 2000]

• **unquenched NRQCD** \(n_f = 2\)
  
  \[ B^{s}_1 = 0.85(2)(6), \quad B^{s}_2 = 0.84(6)(8) \]

  [JLQCD (S. Aoki et al.), 2001-2003]

• **QCD** \((m_c \ll m_Q < m_b, m_Q \to m_b)\)
  
  \[ B^{s}_1 = 0.91(3)_{-6}^{+0}, \quad B^{s}_2 = 0.86(2)^{+2}_{-3} \]

  [APE (D. Becirevic et al.), 2000]

• **QCD + HQET**
  
  \[ B^{s}_1 = 0.87(2)(5), \quad B^{s}_2 = 0.84(2)(4) \]

  [APE (D. Becirevic et al.), 2001]
Lifetime Ratios

\[ \frac{\tau(B^+)}{\tau(B_d)} \]

\[ \frac{\tau(B_s)}{\tau(B_d)} \]

\[ \frac{\tau(\Lambda_b)}{\tau(B_d)} \]
### Theoretical predictions at the NLO + contribution of $O(1/m_b^4)$:

\[
\frac{\tau(B^+)}{\tau(B_d)} = 1.06 \pm 0.02, \quad \frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.88 \pm 0.05
\]

[E. Franco, V. Lubicz, F. Mescia and C. T., 2002-2003]

### Experimental measurements:

\[
\frac{\tau(B^+)}{\tau(B_d)} = 1.081 \pm 0.015, \quad \frac{\tau(B_s)}{\tau(B_d)} = 0.939 \pm 0.044, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.803 \pm 0.047
\]

[LEP+CDF+B-factories average, Heavy Flavor Averaging Group (HFAG), 2004]

- Good agreement at the NLO and $O(1/m_b^4)$
- $\frac{\tau(\Lambda_b)}{\tau(B_d)}$ at 1σ

### Status - Lifetime Ratios Experiment, HQE

<table>
<thead>
<tr>
<th>$\tau(B^+) / \tau(B_d)$</th>
<th>$\tau(B_s) / \tau(B_d)$</th>
<th>$\tau(\Lambda_b) / \tau(B_d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>1.01(3)</td>
<td>1.00(1)</td>
</tr>
<tr>
<td>NLO</td>
<td>1.06(3)</td>
<td>1.00(1)</td>
</tr>
<tr>
<td>NLO+ $O(1/m_b^4)$</td>
<td>1.06(2)</td>
<td>1.00(1)</td>
</tr>
</tbody>
</table>

Exp from LEP-B averaging group, with results from LEP, CDF Run I, and, for $\tau_{\tau_{B_d}}$, BaBar & BELLE (fully rec., hadronic).
Width Differences

\[ \frac{\Delta \Gamma_q}{\Gamma_q} = -\frac{\Delta m_q}{\Gamma_q} \text{Re} \left( \frac{\Gamma^q_{21}}{M^q_{21}} \right) \]

\[ (\Gamma^q_{21}/M^q_{21} = \mathcal{O}(m_b^2/m_t^2), \Delta \Gamma_d/\Delta \Gamma_s = \mathcal{O}(\lambda^2)) \]

NLO distr. vs LO distr.

\[ (\Delta \Gamma_d/\Gamma_d) 10^3 \]

\[ (\Delta \Gamma_s/\Gamma_s) 10^2 \]
Experimental measurements:

\[ \left| \frac{\Delta \Gamma_d}{\Gamma_d} \right| = 0.008 \pm 0.037 \pm 0.018 \] (BaBar collaboration, 2003)

\[ \frac{\Delta \Gamma_s}{\Gamma_s} = 0.07^{+0.09}_{-0.07} \] (HFAG, 2004)

Theoretical predictions at the NLO + contribution of \( O(1/m_b^4) \):

\[ \frac{\Delta \Gamma_d}{\Gamma_d} = (2.42 \pm 0.59) \times 10^{-3} \]

\[ \frac{\Delta \Gamma_s}{\Gamma_s} = (7.4 \pm 2.4) \times 10^{-2} \]

Waiting for D0 result!!!!!!!!!!!!!
Conclusions

Importance of NLO corrections in QCD and \( O(1/m_b^4) \) in the HQE

Lifetime Ratios

\[
\frac{\tau(\Lambda_b)}{\tau(B_d)} \text{ at } 1\sigma
\]

Width Differences

[CDF, 2004]

\[
\frac{\Delta \Gamma_s}{\Gamma_s} = 0.65^{+0.25}_{-0.33} \pm 0.01 \approx 2\sigma
\]

Waiting for D0 result!