Exploring $B$ Physics via Heavy Quark Expansion in $B \to X_c \ell \nu$

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• Inclusive semileptonic $b \rightarrow c \ell \nu$ distributions can and should be further explored.

• We have good control over inclusive $B \rightarrow X_s + \gamma$ and $B \rightarrow X_u \ell \nu$ decay characteristics with the today’s experimental capabilities, if fully use the potential of the OPE, in particular utilizing OPE fit results from $B \rightarrow X_c \ell \nu$.

• Inclusive studies shed light on $B$ physics in general, including the dynamics driving their specific decays.

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All available clv moments plus the BELLE bsg moments at 1.8 GeV
and bsg CLEO at 2.0 and BABAR moments at 1.9 GeV
ALL BSG moments are fitted with BIAS correction "active"
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FCN= 18.23169 FROM MINOS STATUS=SUCCESSFUL 921 CALLS 1464 TOTAL
EDM= 0.17E-07 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER PARABOLIC MINOS ERRORS
NO. NAME VALUE ERROR NEGATIVE POSITIVE
1 Vcb 0.41444E-01 0.43241E-03 -0.43271E-03 0.43237E-03
2 mb 4.5905 0.40896E-01 -0.40738E-01 0.41060E-01
3 mc 1.1396 0.61849E-01 -0.62339E-01 0.61187E-01
4 mu_pi 0.40798 0.39066E-01 -0.39659E-01 0.38547E-01
5 rho_d 0.79222E-01 0.24313E-01 -0.24551E-01 0.24087E-01
6 mu_g 0.29860 0.48423E-01 -0.49409E-01 0.47610E-01
7 rho_ls -0.18362 0.88020E-01 -0.88044E-01 0.88210E-01
8 alpha 0.30000 constant
9 BR 0.10627 0.13702E-02 -0.13688E-02 0.13719E-02
10 D 0.22000 constant
11 SCALE 1.0000 constant
Why moments? Has been partially addressed in the rest – in the talk

Now when the 1% threshold for $V_{cb}$ has been met, we must work hard to justify the accuracy

Interesting physics lies not only in $V_{cb}$

Precision values of $m_b, m_c$ may eventually test FSM* as precisely as CKM does SM

Any correct treatment in the consistent scheme would yield the same numerical result for $V_{cb}$ within its accuracy

Q: Can we do without adopting a scheme at all?
A: Not while doing calculations. Can display the results in the ‘observable through observable’ form, but in the era of computers this hardly saves time. But we would lose lots of physics cf. end of the talk

Why ‘kinetic scheme’?

Calculations have been done correctly, including (but not limited to) perturbative corrections do not need to call $\alpha_s^2$ terms $\alpha_s^1$, etc.

Theoretically sound /completely formulated

OPE is done according to Wilson
reduced number of new HQ parameters

$m_b(\mu), \mu_\pi^2(\mu), \rho_D^3(\mu)$... are well-defined and have direct physics meaning their values are constrained lead to informative relations e.g. $\mu_\pi^2 \geq \mu_G^2$

*Future Standard Model
Do not need to impose mass constraints on $m_b - m_c$ (though this always remains an option)

Free from expanding in $1/m_c$, even if it is $\frac{8}{m_cm_b}$

All this helps making substantiated error estimates
many alternative suggestions are difficult to accept

Calculations are quite stable, show mild cut-dependence; minimal scale-dependence little time-dependence

Running coupling resummation (BLM) is not a problem:

With the IR piece cut off according to Wilson we can work for precision!

Can the similar analysis be done in other approaches?

So, calculate all the observables in terms of $m_b(1\text{ GeV}), m_c(1\text{ GeV}), \mu_\pi^2(1\text{ GeV}), \mu_G^2, \rho_D^3, \rho_{LS}^3, \ldots$

and confront with the experimental data

Aquila, P.G., Ridolfi, N.U. hep-ph/0503083

Opinion: Fits should be left to professionals (experiments) theorists provide only logistic support
A comprehensive fit including all moment measurements:
(by the professionals)

Experimental fit to all these data:

\[ V_{cb} = (4.144 \pm 0.043) \cdot 10^{-3} \]

Not all the sources of theory errors are possibly included
• SL decays yielded accurate $m_b$ itself not obvious a priori

The combination $m_b - 0.74 m_c$ is determined with only a 15 MeV error bar!

Running ‘kinetic’ mass is an observable and has no intrinsic limitation on precision

Theoretical expectation: $m_b(1\text{ GeV}) = (4.57 \pm 0.06) \text{ GeV}$

Voloshin 1995–1996
Melnikov, Yelkhovsky 1998–1999
Beneke, Signer

$e^+ e^- \to \gamma(1S, 2S, 3S, 4S, 5S)$
moments of $\sigma(e^+ e^- \to b\bar{b})$
What is still missing:

- $\alpha_s$-corrections to the power-suppressed Wilson coefficients: the principal limiting factor

  S. Gardner, 10/2001

- Is charm sufficiently heavy? we do not expand in $\frac{1}{m_c}$, yet

  Effects of the nonperturbative four-quark expectation values with charm $\langle B|\bar{b}c\bar{c}b|B\rangle$ loosely referred to as ‘Intrinsic Charm’

  Required in the consistent OPE

  see Benson et al., hep-ph/0302262

**Analysis** (Zwicky et al., to appear):

Some surprises in the $1/m$ expansion. The effect appears at the sub-% level in $\Gamma_{sl}$, is expected below 0.5% due to cancellations

Experiment directly constrains the effect at 1 to 2% level

Expect improvement down to 0.5% where it would not affect precision of $V_{cb}$
• No apparent problem with $\langle M_X^2 \rangle$ vs. $E_{\text{cut}}^\ell$

Robust OPE approach à la Wilson, $\mu = 1\text{GeV}$:

- Bigi, N.U. hep-ph/0308165
- Gambino, N.U. hep-ph/0401063

Data and expectations as of July 2003

Second mass moment $\langle [M_X^2 - \langle M_X^2 \rangle]^2 \rangle$:

- Gambino, N.U. hep-ph/0401063
- N.U. hep-ph/0403166

Parameters fixed from the BaBar fit hep-ex/0404017

Good agreement where the right theory is used right
OPE seems to work even where may be expected to break down
Have an accurate and reliable determination of many HQ parameters from experiment

Extracting $|V_{cb}|$ from $\Gamma_{s1}(B)$ has good accuracy and solid grounds

Have precision checks of the OPE at the nonperturbative level

Overall there are many remarkable agreements with predictions

I think the most impressive is good consistency between $\langle M_X^2 \rangle$ and $\langle E_\ell \rangle$: A sensitive check of the nonperturbative sum rule for $M_B - m_b$

Important: the HQ values emerge in accord with the theoretical expectations: $m_b$, $\mu_\pi^2 > \mu_G^2$, ...

the right scale for $\rho_D^3$

Theory seems to work too well?

‘Theoretical correlations’

Need to check in a different environment:

consider $b \rightarrow$ light $q$ decays

Local duality and its violations

Validity of the OPE itself was questioned under ‘duality’ term

Sometimes is regarded as an experiment-only accessible issue

then one should not forget about a successful prediction

it must be very small in inclusive rates at low cut
• \( b \to s + \gamma \) moments?

Problems were faced when relied on relations *imprecise* with a high cut on \( E_\gamma \)

\[
\langle E_\gamma \rangle = \frac{m_b}{2} + \ldots \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = \frac{\mu^2}{12} + \ldots
\]

A good way to accurately measure HQ parameters...

**Bottle neck:** ‘Hardness’ \( Q \) often gets too low with the cuts

even in \( b \to c \ell \nu \) \( Q \simeq m_b - m_c \) for total widths, but

\( Q \) is below 1 GeV for \( E_\ell > 1.7 \) GeV

A complementary consideration suggests the expansion for \( M_X^2 \) loses sense

for \( E_{\text{cut}} \geq 1.7 \) GeV

Terms appear \( \propto e^{\frac{-Q}{\mu_{\text{hadr}}}} \)

In \( b \to s + \gamma \) \( Q \simeq M_B - 2E_{\text{min}} \simeq 1.2 \) GeV

if the cut is at \( E_\gamma = 2 \) GeV

Accounting for these biases yielded a good agreement between all measurements
Perturbative corrections with the explicit Wilsonian cutoff have been calculated including all orders in BLM

Benson, Bigi, N.U. hep-ph/0410080

BELLE 2004: With $E_\gamma > 1.8$ GeV cut biases are not that much an issue

$$\langle E_\gamma \rangle = 2.292 \pm 0.026_{\text{stat}} \pm 0.034_{\text{sys}} \text{ GeV}$$

$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0305 \pm 0.0073_{\text{stat}} \pm 0.0063_{\text{sys}} \text{ GeV}^2$$

For the extracted HQ values we would get

$$\langle E_\gamma \rangle = 2.315 \text{ GeV}$$

$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0325 \text{ GeV}^2$$

CLEO 2001: $E_{\text{cut}} = 2$ GeV

$$\langle E_\gamma \rangle = 2.346 \pm 0.032_{\text{stat}} \pm 0.011_{\text{sys}} \text{ GeV}$$

$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0226 \pm 0.0066_{\text{stat}} \pm 0.0020_{\text{sys}} \text{ GeV}^2$$

vs.

$$\langle E_\gamma \rangle = 2.345 \text{ GeV}$$

$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.022 \text{ GeV}^2$$

Quite consistent!

most recent (available), BaBar 2005: $E_{\text{cut}} = 1.9$ GeV

$$\langle E_\gamma \rangle = 2.343 \pm 0.053_{\text{stat}} \pm 0.053_{\text{sys}} \text{ GeV}$$

$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0325 \pm 0.016_{\text{stat}} \pm 0.011_{\text{sys}} \text{ GeV}^2$$

vs.

$$\langle E_\gamma \rangle = 2.327 \text{ GeV}$$

$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0275 \text{ GeV}^2$$
unofficial points have been removed
NB: We need to come up with the new standard for charm mass! For the $\overline{\text{MS}}$ quark mass the normalization around $1.2 \text{ GeV}$ is manifestly too low for precision physics. Even light quarks are nowadays normalized at $2 \text{ GeV}$ $\bar{m}_c(\bar{m}_c)$ is not a stable quantity.

A spectrum of possibilities includes $\bar{m}_c(2\bar{m}_c)$, $\bar{m}_c(m_b)$, $\bar{m}_c(2 \text{ GeV})$, $\bar{m}_c(3 \text{ GeV})$, $\bar{m}_c(5 \text{ GeV})$, ...

Why don’t we come up with a joint recommendation?

My suggestions would be a fixed-scale $\bar{m}_c(2.5 \text{ GeV})$ or $\bar{m}_c(3 \text{ GeV})$
Besides $V_{cb}$ and $V_{ub}$, how do we benefit from knowing the heavy quark parameters?

Precise values of $m_b$, $m_c$ — for textures (future)

Today’s use: $B$ nonperturbative dynamics

OPE operating in terms of the universal QCD operators has comprehensive applications not limited to only inclusive decay rates

For instance, heavy quark sum rules — particularly constraining with the HQP values coming from experiment

advantage of the ‘SV’ (‘kinetic’) renormalization scheme — all bounds are most apparent are not applicable for poorly defined objects

Good example: bound $q^2 > \frac{3}{4}$ N.U. 2000

If $\mu^2_\pi$ is close to $\mu^2_G$ there is also a strong upper bound for the IW slope! N.U. 2002

Assuming the spin sum rule is saturated at $\mu = 1$ GeV we have

$$\mu^2_\pi - \mu^2_G = 3 \varepsilon^2 \cdot (q^2 - \frac{3}{4})$$

Quite a constraint: $$(q^2 - \frac{3}{4}) = \frac{\mu^2_\pi - \mu^2_G}{3\varepsilon^2} \lesssim 0.2 \ (0.4)$$

at $\mu^2_\pi = 0.42 \ (0.5)$ GeV$^2$ since $\varepsilon > 0.35$ GeV

$q^2$ is important in extrapolating the exclusive amplitudes to zero recoil
Eventually this prediction is being confirmed by experiment (?)

One of the miracles of the proximity to the ‘BPS’ regime

\[ \mu_\pi^2 \simeq \mu_G^2 \] is a special point for \( B \) and \( D \) mesons!

In the strict limit \( q^2 = \frac{3}{4} \)

Ultrarelativistic light cloud – antipode to NR quark models

Another application – \( B \to D \ell \nu \): expanding in \( \mu_\pi^2 - \mu_G^2 \) and using the analogue of the Ademollo-Gatto theorem which holds for the BPS expansion

\[
\frac{M_B + M_D}{2 \sqrt{M_B M_D}} f_+(0) = 1.04 \pm 0.01 \pm 0.01
\]

All orders in \( 1/m \) in ‘BPS’, to \( 1/m^2 \cdot 1/\text{BPS}^2 \), \( \alpha_s^1 \)
A “$\frac{1}{2} > \frac{3}{2}$” puzzle

Heavy Quark Sum Rules + the known size of $\mu_G^2$, now also of $\mu_\pi^2$ give much information

Spin sum rules strongly suggest that $\frac{3}{2}$ $P$-wave states must dominate over $\frac{1}{2}$ states. This automatically happens in all quark models respecting QCD and Lorentz covariance

Theory:

The most natural solution of all HQSRs:

$\frac{3}{2}$ states at $\epsilon_{\frac{3}{2}} \approx 450$ MeV and $\tau_{\frac{3}{2}} \approx 0.3$ while

$\tau_{\frac{1}{2}} \approx 0.07 \div 0.12$ with $\epsilon_{\frac{1}{2}} \approx 300 \div 500$ MeV

Why?

Average $P$-wave excitation mass gap:

$\tilde{\epsilon}_P \simeq \frac{2\mu_\pi^2}{3\Lambda} \approx 0.45$ GeV

$\sqrt{\frac{\mu_\pi^2}{3(\rho^2-\frac{1}{4})}} \approx 0.45$ GeV

Typical $\tau^2$:

$\tau^2 \simeq \frac{1}{3} (\rho^2-\frac{1}{4}) \approx 0.25$

$\frac{\Lambda}{6 \tilde{\epsilon}_P} \approx 0.25$

and $\tau_{\frac{1}{2}}^2 \ll \tau_{\frac{3}{2}}^2$ from the spin sum rules

Experiment: $\frac{3}{2}$ charm $P$-wave states are narrow and well identified, $\{D_1, D_2^*\}$
Nonleptonic decays $B \rightarrow D^{**}\pi$ assume factorization

While possibly consistent for $\tau_{3/2}$, they used to give definitely too large $\tau_{1/2} \sim 0.4$ (even at $q^2 = 0$), based on $B^- \rightarrow D^{**}\pi^-$

This would contradict spin sum rules

BELLE-CONF-0460: $B^0 \rightarrow D^{**}\pi^-$ modes witness smallness of $\tau_{1/2}$. The data are consistent with the decays only into $\frac{3}{2}$-states, at the right rate!

Theory predictions from the HQ sum rules seem to be confirmed

All the consequences can, probably, be formulated in terms of measurable moments and cross sections only, without recourse to quark masses, nonperturbative heavy quark expectation values, and even to $\alpha_s$

What seems impossible is to come up with these nontrivial connections, at first glance (second either) referring to very different phenomena