Discussion on $V_{cb}$ inclusive

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We all agree that

✓ HQE fits describe moments data very well
✓ Duality violation is strongly constrained
✓ We NOW extract $|V_{cb}|$ with ~2% accuracy (or better)

do we?
What’s next?

Experimental improvements at Y(4S)

- lower $E_l$ cut (down to? ways to improve precision at low $p^*$?)
- larger statistics
- better implementation of radiative effects
  needs th input (monday discussion)

Can we expect a significant reduction of exp errors
or we have reached the ultimate precision?
What’s next (EXP)

- set priorities: hadronic moments?
- measure moments at higher $p^*$ cuts
- measure higher moments (third at least)
- measure modified hadronic moments
  (a combination of $M_x$ and $E_x$ moments, Uraltsev)
- combine measurements studying common exp systematics (HFAG?)
- ...
Theoretical errors

- uncalculated pert corrections $O(\alpha^2)$
- missing power corrections $O(\Lambda^4/m_b^4)$ or $O(\Lambda^4/m_b^2m_c^2)$
- uncalculated pert corrections to Wilson coeff (mixed corrections)
- intrinsic charm
- duality violation
- ...
Theoretical errors are crucial
Groups have different recipes:

- $\pm 0.2 \, \mu_G^2, \mu_\pi^2$
- $\pm 0.3 \, \rho_D^3, \rho_{LS}^3$
- $m_b \pm 20 \, \text{MeV}$
- $m_c \pm 20 \, \text{MeV}$
- $\alpha_s \pm 0.04$
- $\mu$ (wilsonian cutoff~ 1GeV) dependence
- additional 0.7% on rate (IC)
- correlated for each mom

hadronic momts:
$\sigma_n^2(\text{th})=(A \, f_n \, m_B^{2n})^2+(B_n/2)^2$

leptonic, rad. momts:
$\sigma_n^2(\text{th})=(A \, f_n \, (m_B/2)^n)^2+(B_n/2)^2$

- $f_n=1, 1, 1/4, 1/6 \sqrt{3}$
- $A=0.001$
- $B_n$ size of last pert term
- th correlation=exp correlation

Gambino-Ural'tsev, Benson et al, Babar

Bauer et al
Schemes

- Perturbative schemes for quark masses (1S, kinetic, PS, ...)
- Use of HQET relation for $m_b - m_c$; introduces a $1/m_c$ expansion, leads to smaller errors
- Fit $\mu_G^2$ or not
- Many small differences in implementation
- 7 parameters in Bauer et al.
- 6 or 7 parameters in G-U depending what one fits
Scheme dependence to gauge theory error

Bauer et al.
Scale dependence in kinetic scheme within assigned theory errors even if $\mu_b \neq \mu_c$ (preliminary)

FIG. 4: Fit results for $|V_{cb}|$ and $m_0$ in the \textit{kinexp} and \textit{kinuc} schemes using $\mu_b = 1$ GeV (blue and black) and using $\mu_b = 1.5$ GeV (green and yellow). $\mu_c$ for the \textit{kinuc} scheme has been kept fixed at 1 GeV. The regions correspond to $\Delta\chi^2 = 1$ (black and yellow) and 4 (blue and green). The upper plots include theory errors in the fit, and the lower plot does not.
Can we improve the theory errors?

- Perturbative calculations $O(\alpha_s^2)$ and $O(\alpha_s/m_b^2)$
- Intrinsic charm Uraltsev, Zwicky
- ...

Paolo Gambino  CKM2005
Fractional moments $<M_x^{2n+1}>$

- High $M_x$ tail has higher background
- Not described by OPE
- Two ways of estimating them, disagree
- Do we really need them?
- Can’t we simply dispose of them and put a high cut on $M_x$?
Fractional moments

\[ \langle M_X \rangle = \sqrt{\langle M_X^2 \rangle} \left( 1 + \frac{M_X^2 - \langle M_X^2 \rangle}{\langle M_X^2 \rangle} \right) \]

\[ = \sqrt{\langle M_X^2 \rangle} \left( 1 - \frac{3}{8} \frac{H_2}{\langle M_X^2 \rangle} + \ldots \right) \]

expanded at \( \Lambda^3 \)

Buchmuller, Flaecher

Bauer et al

\[ \langle M_X \rangle = \left\langle \sqrt{m_b^2 s_0 + 2m_b \Lambda E_0 + \Lambda^2} \right\rangle \]
How well do we determine $m_b$ and $m_c$?

- $m_b$ determined to 30/50 MeV in the fits, depending on use of HQET relations
- $m_c$ error about 80 MeV ($m_c(m_c)$ very low in 1Sexp scheme)
- $m_b$-$m_c$ error about 10/30MeV $m_b$-.75$m_c$ to 15MeV
- is the $m_c$ determination useful?
- $m_c$ at higher scale helps?
**Comparison with other Determinations**

**Measurements and Predictions of the b-Quark Mass (MS scheme)**

<table>
<thead>
<tr>
<th>Model</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPE: $B \to X_{J^{P}}$</td>
<td>ALEPH 00, OPAL 01, DELPHI 93, Brandenburg et al. 99, Rodrigo, Santamaria, Bilteny 97, Penin, Steinhauser 02, Kuehn, Steinhauser 01, Nason 01, Pineda 01, Hoang 00, Lucha, Schoeberl 00, Beneke, Signer 99, Melnikov, Yelkhovsky 99, Penin, Pivovarov 99, Kuehn, Penin, Pivovarov 98, Jamin, Pich 97, Gimenez, Martinelli, Sachrajda 97</td>
</tr>
</tbody>
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$\bar{m}_b(\bar{m}_b) = 4.22 \pm 0.06$ GeV

$\bar{m}_c(\bar{m}_c) = 1.33 \pm 0.10$ GeV

*Conversion from kinetic mass scheme to MS scheme with hep-ph/9708372, hep-ph/0302262*

*See also report from CKM WS hep-ph/0304132*

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Moriond QCD, 30. March 04, Henning Flächer (RHUL)
Inclusion of other measurements

- Radiative moments: how should we correct for the photon cut? Neubert’s talk, Benson et al.

- $B \to X_u \ell \nu$ (truncated) moments: useful just to validate SF models?