Wilson Coefficients Fits for New Physics Contributions in Rare B Decays

http://ckm2005.ucsd.edu

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Most measurements of the Unitarity Triangle are in good agreement with SM predictions.

One exception may be $\sin 2\beta$ measured in modes dominated by $b\to s$ penguins (e.g. $B\to \phi K^0_S$).

Presently, discrepancy to $\sin 2\beta$ from charmonium modes is $3.7\sigma$.

If discrepancy is caused by New Physics, it should also affect radiative decays.
Radiative Decays

- Radiative decays are described by effective Hamiltonian in OPE

\[ H_{\text{eff}} = - \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu)O_i(\mu) + C_s(\mu)O_s(\mu) + C_p(\mu)O_p(\mu) \]

- I have neglected here contributions from right-handed couplings

- We expect that “sizable” New Physics contributions may appear in dipole couplings with a \( \gamma, g \) or \( t\bar{t} \), \( (O_7, O_8, O_9, O_{10}) \) or in the scalar & pseudoscalar operators \( O_s \) & \( O_p \) absent in SM

- BABAR & Belle have measured various properties of radiative decays with high-statistics B- samples
With scalar & pseudoscalar couplings \( \frac{d\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{ds} \) differential decay rate at NNLO is predicted to be

\[
\frac{d\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{d\hat{s}} = \frac{G_F^2 \alpha^2 (m_{b,\text{pole}})^5}{768\pi^5} \left| V_{tb} V_{ts}^* \right|^2 (1 - \hat{s}) \sqrt{1 - \frac{4m_{\ell}^2}{\hat{s}}} \left[ 1 + \frac{2m_{\ell}^2}{\hat{s}} \right] + 6m_{\ell}^2 \left( \tilde{C}_9^\text{eff}^2 - |\tilde{C}_{10}^\text{eff}|^2 \right)
\]

\[
\times \left\{ \begin{array}{l}
12 \text{Re} \left( \tilde{C}_7^\text{eff} \tilde{C}_9^\text{eff*} \right) + \frac{4 \tilde{C}_7^\text{eff}^2 (2 + \hat{s})}{\hat{s}} \left[ 1 + \frac{2m_{\ell}^2}{\hat{s}} \right] + 6m_{\ell}^2 \left( \tilde{C}_9^\text{eff}^2 - |\tilde{C}_{10}^\text{eff}|^2 \right) \\
\left( |\tilde{C}_9^\text{eff}|^2 + |\tilde{C}_{10}^\text{eff}|^2 \right) \left[ 1 + 2\hat{s} + \frac{2m_{\ell}^2 (1 - \hat{s})}{\hat{s}} \right] + \frac{3}{2} \hat{s} \left[ 1 - \frac{4m_{\ell}^2}{\hat{s}} \right] \tilde{C}_s^2 + \tilde{C}_p^2 \\
+ 6m_{\ell} \text{Re} \left( C_p \tilde{C}_{10}^\text{eff*} \right) + \frac{1}{m_b} \text{corrections} \end{array} \right\}
\]

Hiller & Krüger (hep-ph/0310210)

If we ignore scalar & pseudoscalar couplings, some \( m_\ell \) terms drop.
Effective Wilson Coefficients

- The effective Wilson coefficients are specified in NNLO

\[
\tilde{C}_7^{\text{eff}} = \left[1 + \frac{\alpha_s(\mu)}{\pi} \omega_7(\hat{s})\right] A_7(\mu) - \frac{\alpha_s(\mu)}{4\pi} \sum_{i=1,2} F_i^{(7)}(\hat{s}) C_i^{(0)}(\mu) + F_8^{(7)}(\hat{s}) A_8^{(0)}(\mu)
\]

\[
\tilde{C}_9^{\text{eff}} = \left[1 + \frac{\alpha_s(\mu)}{\pi} \omega_9(\hat{s})\right] \left[ A_9(\mu) + T_9 h(\hat{m}_c^2, \hat{s}) + U_9 h(1, \hat{s}) + W_9 h(0, \hat{s}) \right] - \frac{\alpha_s(\mu)}{4\pi} \sum_{i=1,2} F_i^{(9)}(\hat{s}) C_i^{(0)}(\mu) + F_8^{(9)}(\hat{s}) A_8^{(0)}(\mu)
\]

\[
\tilde{C}_{10}^{\text{eff}} = \left[1 + \frac{\alpha_s(\mu)}{\pi} \omega_9(\hat{s})\right] A_{10}(\mu)
\]

- The \( A_i \) are constructed from the appropriate \( C_i \)
B→X_sγ Decay Rate

- Inclusive B→X_sγ decay rate at NLO is predicted to be

\[ \Gamma(B \rightarrow X_s\gamma) = \frac{G_F^2 \alpha(m_{b,\text{pole}})^3 m_b^2}{32\pi^4} V_{tb} V_{ts}^* |V_{tb} V_{ts}^*|^2 \left( |P(E_0)|^2 + B(E_0) + N(E_0) \right) \]

- Perturbative Bremsstrahlung non-perturbative
  - \( b \rightarrow s\gamma, b \rightarrow s\gamma qq \)
  - \(~5\% @E_0 > 1.8 \text{ GeV}\)
  - \(<4\% @ 1 < E_0 < 2 \text{ GeV}\)

- The perturbative part is specified in terms of c/u loops, t loop & EW correction

\[ P(E_0) = K_c + \left( 1 + \frac{\alpha_s(\mu_0)}{\pi} \ln \frac{\mu_0^2}{m_t^2} \right) r(\mu_0) K_t + \varepsilon_{EW} \]

- First 2 terms in \( P(E_0) \) involve \( C_7 \) and via operator mixing also \( C_8 \)
B→X_s g Decay Rate

- Inclusive B→X_s g decay rate at NLO is predicted to be

\[ \Gamma(B \rightarrow X_s g) = \frac{G_F^2 \alpha_s(\mu_b)(m_{b,pole})^5}{24\pi^4} \left| V_{tb} V_{ts}^* \right|^2 \tilde{D}^2 + \Gamma_{\text{brems}}^{\text{fin}} \]

- \( D \) is dominated by \( C_8 \)

\[ \bar{D} = C_{8,\text{eff}}^0 + \frac{\alpha_s(\mu_b)}{4\pi} \left\{ C_{8,\text{eff}}^1 - \frac{16}{3} C_{8,\text{eff}}^0 + \sum_{i=1,2} C_i^0 \left[ \ell_i \ln \frac{m_b}{\mu_0} + r_i \right] \right\} \]

\[ + C_{8,\text{eff}}^0 \left[ (\ell_8 + 8 + \beta_0) \ln \frac{m_b}{\mu_0} + r_8 \right] \]

Bremsstrahlung correction

Greub & Liniger (hep-ph/0009144)
For $B \rightarrow X_s \gamma$ the CP asymmetry is predicted as

$$A_{CP}(B \rightarrow X_s \gamma) = \frac{\alpha_s(\mu_b)}{C_{7}^{\text{eff}}} \frac{40}{81} \Im \left[ C_2 C_7^{* \text{eff}} \right]$$

$$- \frac{8}{9} z \left[ V(z) + b(z, \delta) \right] \Im \left[ \left( 1 + \frac{V^* V_{us}}{V_{ts}^* V_{tb}} \right) C_2 C_7^{* \text{eff}} \right]$$

$$- \frac{4}{9} \Im \left[ C_8 C_7^{* \text{eff}} \right] + \frac{8}{27} zb(z, \delta) \Im \left[ \left( 1 + \frac{V^* V_{us}}{V_{ts}^* V_{tb}} \right) C_2 C_8^{* \text{eff}} \right]$$


Here, we explicitly sample the interference term between $C_7$ & $C_8$
For $B \rightarrow X_s \ell^+ \ell^-$ the CP asymmetry in LO is predicted as

$$A_{CP}(B \rightarrow X_s \ell^+ \ell^-) = \frac{\Im \left[ (V^* V_{ts} V^* V_{tb}) D_i^{(s)} \right]}{|V_{cb}|^2 \Delta B_{s\ell\ell}}$$


The functions $D_i$ contain the Wilson coefficients $C_7^{eff}, C_9^{eff}, C_{10}^{eff}$

Here, we sample the interference terms between $C_7$ & $C_8$, $C_7$ & $C_9$ and $C_8$ & $C_9$

For the global fits the above function needs to be reexpressed in NNLO with the same functions used in the branching fraction
In first round of fits include new physics contributions in
\( C_7, C_8, C_9, C_{10} \) (8 new parameters)

Consider leading order effects for the moment at scale \((\mu_0)\)

- \( C_7^{(0)} (\mu_0) \rightarrow C_7^{(0)} (\mu_0) + C_7^{NP} (\mu_0) \)
- \( C_8^{(0)} (\mu_0) \rightarrow C_8^{(0)} (\mu_0) + C_8^{NP} (\mu_0) \)
- \( C_9^{(0)} (\mu_0) \rightarrow C_9^{(0)} (\mu_0) + C_9^{NP} (\mu_0) \)
- \( C_{10}^{(0)} (\mu_0) \rightarrow C_{10}^{(0)} (\mu_0) + C_{10}^{NP} (\mu_0) \)

Run them down to scale \( \mu_b \), \( \Rightarrow \) due to mixing \( C_i^{NP}(\mu_b) \) enter in different \( C_{\text{eff}} \), e.g. \( C_8^{NP} \) in \( C_7^{\text{eff}} \) & \( C_9^{\text{eff}} \)

Include \( C_5 (\mu_0) \) & \( C_6 (\mu_0) \) at later stage
Consider ratios of observables, e.g. $\frac{B(B \to X_s \tau\bar{\tau})_{\Delta m_{\text{ll}} \text{ bin}}}{B(B \to X_s \tau\bar{\tau})_{\text{tot}}}$
- CKM parameters drop out
- $m_b$, pole drops out
- Don't need $\text{B} \to X_c \ell\nu$ as normalization

Consider appropriate ratios of following observables for 1st round:
- $B(B \to X_s \tau\bar{\tau})$, $B(B \to X_s \gamma)$, $B(B \to X_s g)$, $A_{\text{CP}}(B \to X_s \gamma)$, $A_{\text{CP}}(B \to X_s \tau\bar{\tau})$
- get 11 observable to determine 8 parameters

Perform maximum likelihood fit using frequentist approach

Treat all correlations among measured quantities ($m_t$, ...)

Include all experimental errors

Scan over all theoretical uncertainties
In the BABAR analysis we show $\frac{d\mathcal{B}(B\to X_s \ell^+ \ell^-)}{ds}$ in 5 $m_{\ell\ell}$ bins while Belle uses 4 bins in $q^2$.

Since first and last bins are identical we can average $\frac{d\mathcal{B}(B\to X_s \ell^+ \ell^-)}{ds}$ here.

For the other bins we use BABAR & Belle results separately.
<table>
<thead>
<tr>
<th>Observable</th>
<th>Present data sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(B \to X_s \mu^+ \mu^-)$, $0.2 \leq m_{\mu\mu} &lt; 1$ GeV</td>
<td>$(0.45\pm0.48)\times10^{-6}$ BABAR/Belle</td>
</tr>
<tr>
<td>$B(B \to X_s \mu^+ \mu^-)$, $1 \leq m_{\mu\mu} &lt; 2$ GeV</td>
<td>$(1.6\pm0.6\pm0.5)\times10^{-6}$ BABAR</td>
</tr>
<tr>
<td>$B(B \to X_s \mu^+ \mu^-)$, $2 \leq m_{\mu\mu} &lt; m_{\mu\mu}$ GeV</td>
<td>$(1.8\pm0.6)\times10^{-6}$ BABAR</td>
</tr>
<tr>
<td>$B(B \to X_s \mu^+ \mu^-)$, $m_{\mu\mu} \leq m_{\mu\mu} &lt; 5$ GeV</td>
<td>$(0.79\pm0.30)\times10^{-6}$ BABAR/Belle</td>
</tr>
<tr>
<td>$B(B \to X_s \mu^+ \mu^-)$, $2 \leq m_{\mu\mu} &lt; 2.44$ GeV</td>
<td>$(1.49\pm0.50\pm0.33)\times10^{-6}$ Belle</td>
</tr>
<tr>
<td>$B(B \to X_s \mu^+ \mu^-)$, $2.44 \leq m_{\mu\mu} &lt; 3.79$ GeV</td>
<td>$(0.73\pm0.61\pm0.17)\times10^{-6}$ Belle</td>
</tr>
<tr>
<td>$B(B \to X_s \mu^+ \mu^-)$</td>
<td>$(4.83\pm0.81_{\text{exp}}\pm1.18_{\text{th}})\times10^{-6}$ BABAR/Belle</td>
</tr>
<tr>
<td>$B(B \to X_s g)$</td>
<td>$0.017\pm0.043$ CLEO</td>
</tr>
<tr>
<td>$B(B \to X_s \gamma)$</td>
<td>$(3.45\pm0.24\pm0.15)\times10^{-4}$ all</td>
</tr>
<tr>
<td>$A_{CP}(B \to X_s \mu^+ \mu^-)$</td>
<td>$-0.22\pm0.26\pm0.02$ BABAR</td>
</tr>
<tr>
<td>$A_{CP}(B \to X_s \gamma)$</td>
<td>$-0.109\pm0.116$ BABAR/Belle</td>
</tr>
<tr>
<td>$A_{FB}(B \to X_s \mu^+ \mu^-)$</td>
<td>No data</td>
</tr>
</tbody>
</table>
**Present & Future Measurement Inputs**

- **Consider following ratios of branching fractions & $A_{CP}$**
  
  \[
  \frac{\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)}{\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\Delta m_{\text{bin}}}} / \mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)_{\text{tot}} \quad \frac{\mathcal{B}(B \rightarrow X_s \gamma)}{\mathcal{B}(B \rightarrow X_s g)} \]

  \[
  \frac{\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)}{\mathcal{B}(B \rightarrow X_s \gamma)} \]

- **Include other measurements, e.g. exclusive decays later as well as add $C_S$ & $C_P$ to the fit**

  \[
  \frac{\mathcal{B}(B \rightarrow X_s \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow X_s e^+ e^-)} \quad \frac{\mathcal{B}(B \rightarrow K^* \gamma)}{\mathcal{B}(B \rightarrow X_s \gamma)} \]

  \[
  \frac{\mathcal{B}(B \rightarrow K \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow K e^+ e^-)} \quad \frac{\mathcal{B}(B \rightarrow K^* \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow K^* e^+ e^-)} \]

  \[
  \frac{\mathcal{B}(B \rightarrow K \ell^+ \ell^-)}{\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)} \quad \frac{\mathcal{B}(B \rightarrow K^* \ell^+ \ell^-)}{\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-)} \]

  \[
  A_{CP} (B \rightarrow K \ell^+ \ell^-) \quad A_{CP} (B \rightarrow K^* \ell^+ \ell^-) \]

  \[
  A_{FB} (B \rightarrow K^* \ell^+ \ell^-) \quad A_{CP} (B \rightarrow K^* \gamma) \]

  \[
  \sin 2\beta (B \rightarrow \phi K_S^0) \]

- **This adds another 11 measurements & 4 more parameters $C_S, C_P$**
Theoretical Parameters

- There are several theoretical uncertainties that will be accounted for by scanning over them.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$ scale for charm loop</td>
<td>40 - 120 GeV</td>
</tr>
<tr>
<td>$\mu_0$ scale for top loop</td>
<td>120 - 240 GeV</td>
</tr>
<tr>
<td>$\mu_b$ scale</td>
<td>2.5 - 10 GeV</td>
</tr>
<tr>
<td>$m_c/m_b$</td>
<td>0.23 - 0.29</td>
</tr>
<tr>
<td>$1/m_b$ corrections</td>
<td>?</td>
</tr>
<tr>
<td>theory uncertainty in $B \rightarrow X_s\gamma$</td>
<td>$\pm 1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>theory uncertainty in $B \rightarrow X_s\ell^+\ell^-$</td>
<td>$\pm 1.18 \times 10^{-6}$</td>
</tr>
<tr>
<td>theory uncertainty in $B \rightarrow X_s g$</td>
<td>?</td>
</tr>
</tbody>
</table>

There may be other theory uncertainties that need to be included ($\Delta \sim \Lambda_{QCD}$?)

Take as 100% correlated
Fit Procedure

- A $\chi^2$ minimization is performed with a particular set of theoretical parameters

\[
\chi^2_{\text{set}}(C_7^{NP}, C_8^{NP}, C_9^{NP}, C_{10}^{NP}) = \sum_n \left( \frac{\langle Y \rangle_n - Y_{\text{set}}(C_7^{NP}, C_8^{NP}, C_9^{NP}, C_{10}^{NP})}{\sigma_{Y,n}} \right)^2
\]

- Here $\langle Y \rangle$ denotes an observable & $\sigma_Y$ represents statistical and systematic errors added in quadrature

- $Y_{\text{set}}$ is the theoretical prediction for the observable for a specific set of the theory parameters ($\mu_0$, $\mu_b$, $m_c/m_b$, ...)

- We fit many individual theory sets scanning over the allowed theoretical parameter space for each of these parameters

- We consider a theory set consistent with data, if $P(\chi^2_{\text{min}}) > 5\%$
  - For these we determine central values of $C_7^{NP}$, $C_8^{NP}$, $C_9^{NP}$, $C_{10}^{NP}$ and plot contours, e.g. $\text{Re}(C_7^{NP})-\text{Im}[C_7^{NP}]$, $\text{Re}(C_7^{NP})-\text{Re}[C_8^{NP}]$, $\text{Re}(C_9^{NP})-\text{Re}[C_{10}^{NP}]$, ...
  - The contours of various theory sets are overlayed
\[ \chi^2(M(C_{7\text{NP}}, C_{8\text{NP}}, C_{9\text{NP}}, C_{10\text{NP}}) = \left( \lambda - \lambda \right)^2 + \left( \frac{m_t - m_t}{\sigma_{m_t}} \right)^2 + \left( \frac{m_c/m_b - m_c/m_b}{\sigma_{m_c/m_b}} \right)^2 + \left( \frac{m_W - m_W}{\sigma_{m_W}} \right)^2 \]

\[ + \sum_{\text{bin}=1}^{7} \left( \frac{B_{\text{meas}}(B \rightarrow X_s \ell^+ \ell^-) / B(B \rightarrow X_s \ell^+ \ell^-) - \Gamma_{\text{th}}^{C_{7\text{NP}}, C_{8\text{NP}}, C_{9\text{NP}}, C_{10\text{NP}}) / \Gamma(C_{7\text{NP}}, C_{8\text{NP}}, C_{9\text{NP}}, C_{10\text{NP}})}{\sigma[B \rightarrow X_s \ell^+ \ell^-] / B \rightarrow X_s \ell^+ \ell^-]_{\text{tot}}} \right)^2 \]

\[ + \left( \frac{B(B \rightarrow X_s \ell^+ \ell^-) / B(B \rightarrow X_s \gamma) - \Gamma_{\text{th}}^{C_{7\text{NP}}, C_{8\text{NP}}, C_{9\text{NP}}, C_{10\text{NP}}) / \Gamma(C_{7\text{NP}}, C_{8\text{NP}}, C_{9\text{NP}}, C_{10\text{NP}})}{\sigma[B \rightarrow X_s \ell^+ \ell^-] / B \rightarrow X_s \gamma]} \right)^2 \]

\[ + \left( \frac{B(B \rightarrow X_s \gamma) / B(B \rightarrow X_s \gamma) - \Gamma_{\text{th}}^{C_{7\text{NP}}, C_{8\text{NP}}} / \Gamma(C_{7\text{NP}}, C_{8\text{NP}})}{\sigma[B \rightarrow X_s \gamma] / B \rightarrow X_s \gamma]} \right)^2 \]

\[ + \left( \frac{A_{CP}(B \rightarrow X_s \gamma) - A_{CP}^{\text{th}}(C_{7\text{NP}}, C_{8\text{NP}})}{\sigma[A_{CP}(B \rightarrow X_s \gamma)]} \right)^2 \]

\[ + \left( \frac{A_{CP}(B \rightarrow X_s \ell^+ \ell^-) - A_{CP}^{\text{th}}(C_{7\text{NP}}, C_{8\text{NP}})}{\sigma[A_{CP}(B \rightarrow X_s \ell^+ \ell^-)]} \right)^2 \]
Correlation Plots

We will determine correlation plots between pairs of Wilson coefficients, similarly as those presented by Hiller & Krüger in their model-independent analysis.

Hiller & Krüger (hep-ph/0310210)
Issues in Exclusive Modes

- Exclusive $b \rightarrow s$ exclusive modes involve form factors introducing large theoretical uncertainties.

- For $B \rightarrow K^{\ast} \ell^\pm \nu$ need 3 form factors: $f_+, f_0, \& f_T$

- For $B \rightarrow K^{\ast \ast} \ell^\pm \nu$ need 7 form factors: $A_0, A_1, A_2, V, T_1, T_2, \& T_3$

- For $B \rightarrow K^{\ast \gamma}$ need 1 tensor form factor $T_1(0)$

- Each form factor is specified by 3-4 parameters that each contains theoretical uncertainties.

- Considering only small $q^2$ values (below $m_\psi$), however, the form factors in $B \rightarrow K^{\ast \ell \nu}$ reduce to 1 independent form factor and those in $B \rightarrow K^{\ast \ell \nu}$ to 2 independent form factors.

- Even in this scenarios there are still too many parameters to vary $\Rightarrow$ so take a representative set of $\sim 10$ parametrizations.
Conclusions

- Model-independent global fits of Wilson coefficients in radiative decays (& other rare processes) to explore New Physics effects provide a rather useful tool & now is the time to develop them.

- It will allow us to study effects from varying different theory parameters.

- Present data will constrain $C_7^{\text{NP}}$, $C_9^{\text{NP}}$, $C_{10}^{\text{NP}}$, that may be already interesting e.g. in $C_7$-$C_8$ plane.

- With increasing data samples, constraints will become improve.

- We can study what you can expect at a sample of 1 $\text{ab}^{-1}$/year.

- We can estimate what $\mathcal{L}$’s are required to find New Physics effects at a 10%, 5% or 1% level (case for Super B).
Status & Outlook

- Development of the fit program is rather advanced

- Independently, all branching fractions have calculated in Mathematica (for cross checks) apart from a technical problem in $\Gamma_{\text{fin}}^{\text{brems}}$

- Need to add CP asymmtries calculations in Mathematica

- Need to finish the fit program & debug it

- After first results proceed to include $C_s$ & $C_p$ terms & other measurements, as well as to use maximum likelihood fit instead of $\chi^2$ minimization

- I am open to new suggestions or criticism

Many thanks to Gudrun Hiller for pointing me to relevant publications and stimulating discussions