Extracting γ from D Dalitz analysis

theory introduction

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Outline

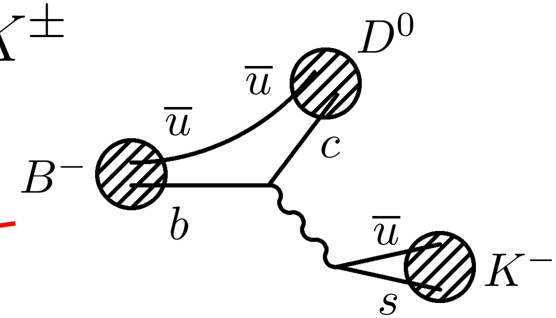
- using fit to Breit-Wigner forms
- model independent method
  - charm factory input
  - errors
- conclusions

Notation

consider the decay chain

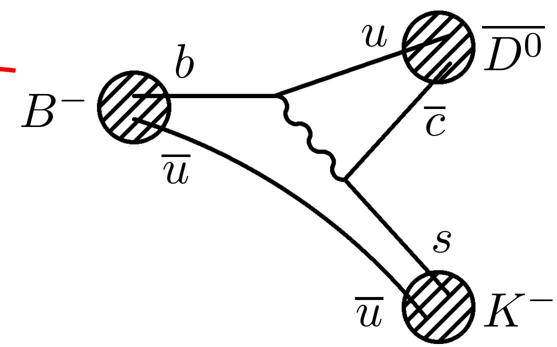
$$B^{\pm} \to DK^{\pm} \to (K_S\pi^-\pi^+)_D K^{\pm}$$

the amplitudes for B decays



$$A(B^- \to D^0 K^-) \equiv A_B$$

$$A(B^- \to \overline{D}^0 K^-) \equiv A_B r_B e^{i(\delta_B - \gamma)}$$



color suppression + CKM $\Rightarrow r_B \sim 0.1 - 0.2$

for three body D decay (CP in D assumed)

$$A_D(s_{12}, s_{13}) \equiv A_{12,13}\, e^{i\delta_{12,13}} \equiv A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3))$$

$$= A(\overline{D}^0 \to K_S(p_1)\pi^+(p_2)\pi^-(p_3))$$

Complete amplitude

$$A(B^- \to (K_S\pi^-\pi^+)_D K^-) =$$

$$A_B \mathcal{P}_D \Big(A_D(s_{12}, s_{13}) + r_B e^{i(\delta_B - \gamma)} A_D(s_{13}, s_{12}) \Big)$$

$$B^- \to D^0 K^- + \overline{D^0} K^-$$

$$A(B^+ \to (K_S\pi^-\pi^+)_D K^+) =$$

$$A_B \mathcal{P}_D \Big(A_D(s_{13}, s_{12}) + r_B e^{i(\delta_B + \gamma)} A_D(s_{12}, s_{13}) \Big)$$

$$B^+ \to \overline{D^0} K^+ + D^0 K^+$$

- need to measure $A(s_{12}, s_{13})$ (also phase)
  - fit to a sum of resonances — Belle coll., 2002
  - model independent approach

Giri, Grossman, Soffer, J.Z.; Atwood, Soni, 2003

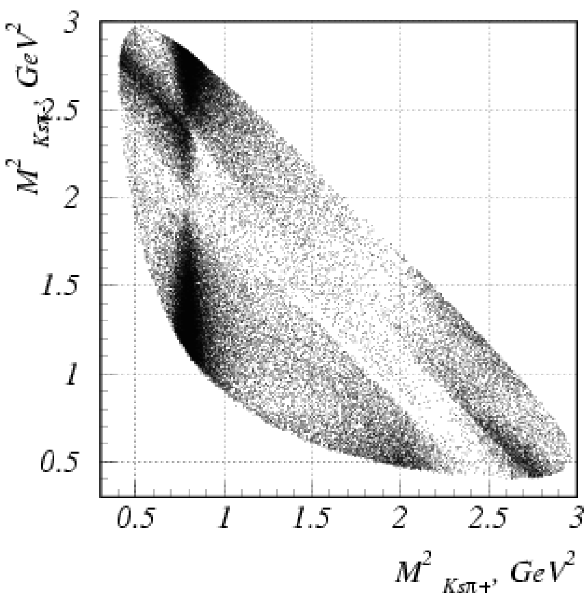
Modeling A_D

model A_D with a fit to a sum of Breit-Wigners:

$$A_D(s_{12}, s_{13}) = A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) =$$

$$= a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12}, s_{13})$$

$$\mathcal{A}_r(s_{12}, s_{13}) = {}^J\mathcal{M}_r \times \frac{1}{s - M_r^2 + iM_r\Gamma_r(\sqrt{s})}$$



- model fit to high statistics D^0 tagged decay data available from B-factories ($N \sim 100k$)
- in $B^\pm \to (K_S\pi^-\pi^+)_D K^\pm$ only r_B, δ_B and γ to be fit

Belle, hep-ex/0308043

Model independent method

- at present the modeling error on γ is estimated to $\sim 10^\circ$
- consider the B^- amplitude

$$A(B^- \to DK^-) = A_B \mathcal{P}_D \big(A_D(s_{12}, s_{13}) + r_B e^{i(\delta_B - \gamma)} A_D(s_{13}, s_{12}) \big)$$

$$B^- \to D^0 K^- + \overline{D^0} K^-$$

$$\hookrightarrow K_s \pi^- \pi^+ \hookleftarrow$$

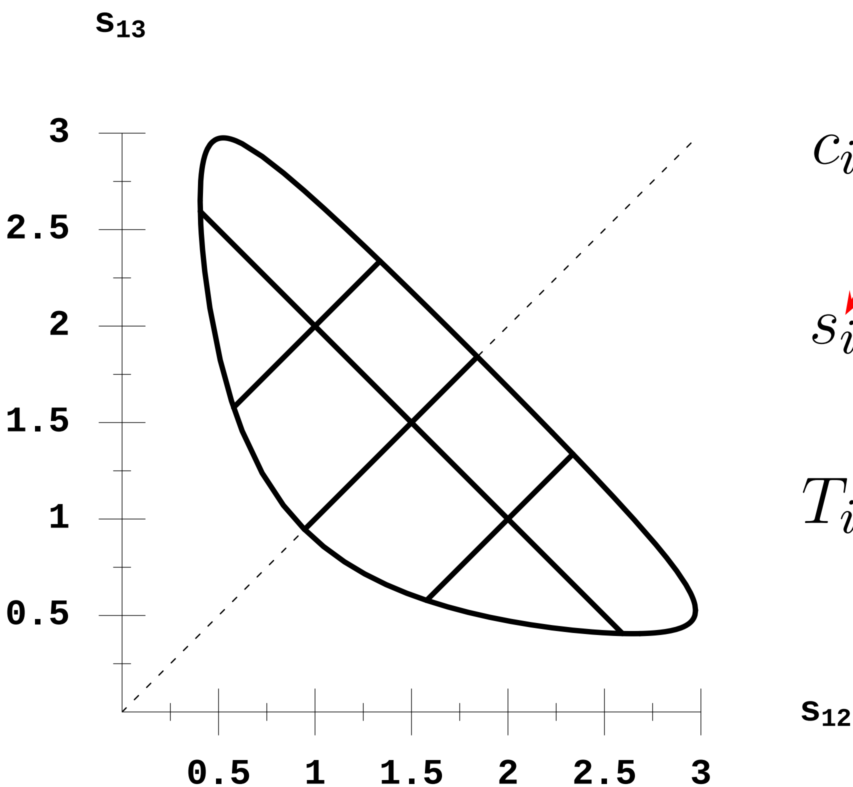
- interference term in the decay width:

$$\mathcal{R}e\big[A_D(s_{12}, s_{13})\, A_D^*(s_{13}, s_{12})\, e^{-i(\delta_B - \gamma)}\big] = A_{12,13}\, A_{13,12} \times$$
$$\big[\cos(\delta_{12,13} - \delta_{13,12}) \cos(\delta_B - \gamma) + \sin(\delta_{12,13} - \delta_{13,12}) \sin(\delta_B - \gamma)\big]$$

unknown strong phases

Model independent method II

- partition Dalitz plot in $2k$ bins
- label bins below symmetry axis i, above axis $\bar{i}$



unknowns

$$c_i \equiv \int_i dp\, A_{12,13} A_{13,12} \cos(\delta_{12,13} - \delta_{13,12})$$

$$s_i \equiv \int_i dp\, A_{12,13} A_{13,12} \sin(\delta_{12,13} - \delta_{13,12})$$

$$T_i \equiv \int_i dp\, A_{12,13}^2$$

measurable from tagged D

using CP :

$$c_{\bar{i}} = c_i, \qquad s_{\bar{i}} = -s_i$$

$s_{12} = m^2_{K_s\pi^-}$ and $s_{13} = m^2_{K_s\pi^+}$

Master formulae

- a set of $4k$ equations
- the k equations for i bins

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) =$$
$$T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i]$$

eqs. for $\hat{\Gamma}_{\bar{i}}^-$, $\hat{\Gamma}_i^+$, $\hat{\Gamma}_{\bar{i}}^+$ obtained by $12 \leftrightarrow 13$ and/or $\gamma \leftrightarrow -\gamma$

- $2k+3$ unknowns: $c_i,\ s_i,\ r_B,\ \delta_B,\ \gamma$

solvable for $k \geq 2$

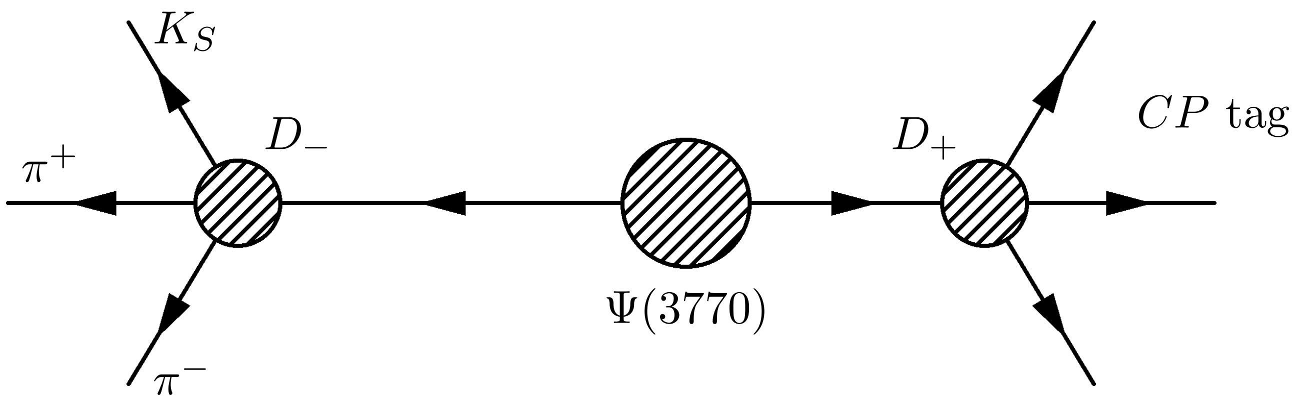
c_i, s_i

- c_i and s_i^2 can be measured at charm factories working at $\psi(3770)$
- they can be bounded using tagged D^0 decays

$$|s_i|,\ |c_i| \le \int_i dp\, A_{12,13}A_{13,12} \le \sqrt{T_i T_{\bar{i}}}$$

Measuring c_i at charm factory

- charm factory, $\psi(3770) \to D\bar{D}$
- use CP eigenstates $D^0_\pm \equiv (D^0 \pm \overline{D}^0)/\sqrt{2}$



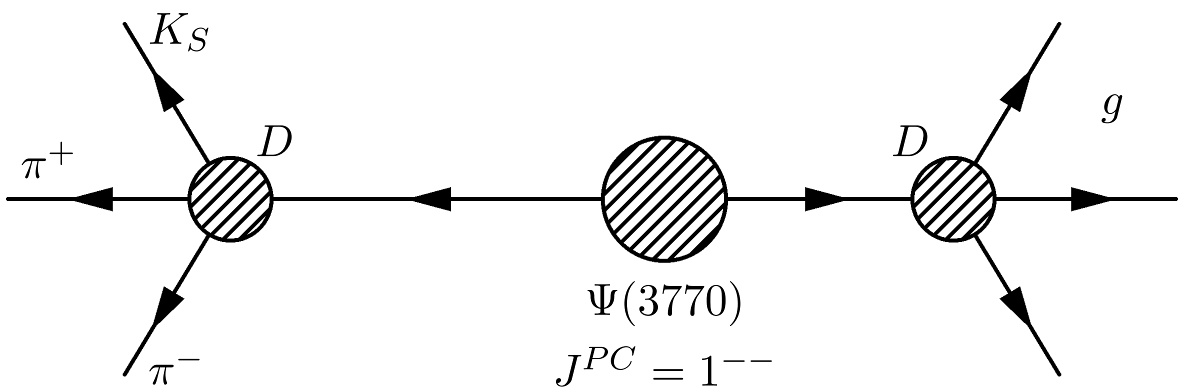
$$d\Gamma(D^0_\pm \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) =$$

$$\frac{1}{2}\left(A^2_{12,13} + A^2_{13,12}\right) \pm A_{12,13}A_{13,12}\cos(\delta_{12,13} - \delta_{13,12})dp$$

$$c_i = \frac{1}{2}\Big[\int_i d\Gamma(D^0_+ \to K_S\pi^-\pi^+) - \int_i d\Gamma(D^0_- \to K_S\pi^-\pi^+)\Big]$$

Measuring s_i^2

- instead of a CP tag consider a decay to general state g



ith bin of $K_S\pi^+\pi^-$ and jth bin of g

$$\Gamma_{i,j} \propto T_i T^g_{\bar{j}} + T_{\bar{i}} T^g_j - 2(c_i c^g_j + s_i s^g_j)$$

- if $g = K_S\pi^+\pi^-$ and $j = i$ ($j = \bar{i}$) one measures s_i^2
- if g a CP even (odd) eigenstate, $s^g_j = 0$, $T^g_j = T^g_{\bar{j}} = \pm c^g_j$, no sensitivity to s_i

Discussion

- formalism extended to $B^0(\bar{B}^0) \to DK_S$ multibody D decays
  Gronau, Grossman, Shumaher, Soffer, J.Z.
- how small need the bins be, not to average out sensitivity to γ?
- CP conserving $D - \bar{D}$ mixing does not change the formalism
- CP violation in D sector the only uncertainty (!)
  - in SM $\lambda^6 \sim 10^{-4}$ suppressed
  - formalism can be extended to include CP viol. in D sector

Including CP viol. in D

- only relevant if beyond SM CP viol. in D
- is it present? compare (time integrated) D^0 and $\bar{D}^0$ Dalitz plots
- CP viol. in $D-\bar{D}$ mixing ($q/p \neq 1$) can be included model independently

$$A(D^0 \to f) = A_f + e^{i\theta} B_f = A_f + \cos\theta B_f + i \sin\theta B_f$$

$$A(D^0(t) \to f) = \overbrace{e^{-\Gamma t/2}\left[A_f - \tfrac{1}{4}\bar{A}_f\left(\tfrac{q}{p} + \tfrac{p}{q}\right)(x+iy)\Gamma t + \cos\theta B_f\right]}^{A_f^{\text{even}}(t)}$$
$$+ e^{-\Gamma t/2}\left[-\tfrac{1}{4}\bar{A}_f\left(\tfrac{q}{p} - \tfrac{p}{q}\right)(x+iy)\Gamma t + i\sin\theta B_f\right]$$

$$\bar{A}_f^{\text{even}}(t) = \exp(-\Gamma t/2)\bar{A}_f + \cdots$$

Including CP viol. in D, cont.

- redefine: $\int_i |A_f^{\rm even}|^2 = T_i$, $\quad \int_i A_f^{\rm even} A_{\bar f}^{\rm even *} = c_i + i s_i$
- partial D decay width

$$\int_i |A(D^0(t) \to f)|^2 = T_i + 2\ sin\theta\ \mathcal{I}m \int_i A_f^{\rm even} B_f^* \\ -\tfrac{1}{2}\mathcal{R}e\Big[\big(\tfrac{q}{p} - \tfrac{p}{q}\big)^*(x+iy)^*(c_i + is_i)\Big]$$

- modified expression for B decay

$$\int_i |A(B^- \to f_D K^-|^2 =$$

$$A_B^2 \Big\{ T_i + r_B^2 T_{\bar i} + 2 r_B [\cos(\delta_B - \gamma) c_i + \sin(\delta_B - \gamma) s_i]$$

$$+ \mathcal{R}e\Big[\big(\tfrac{q}{p} - \tfrac{p}{q}\big)^*(x+iy)^* F(c_i, s_i, T_i, T_{\bar i}, r_B, \delta_B, \gamma)\Big]$$

$$+ 2 \sin\theta \mathcal{I}m\Big[\big(A_f^{\rm even} + r_B e^{i(\delta_B - \gamma)} \bar A_f^{\rm even}\big)\big(B_f - r_B e^{i(\delta_B-\gamma)} \bar B_f\big)^*\Big]$$

Conclusions

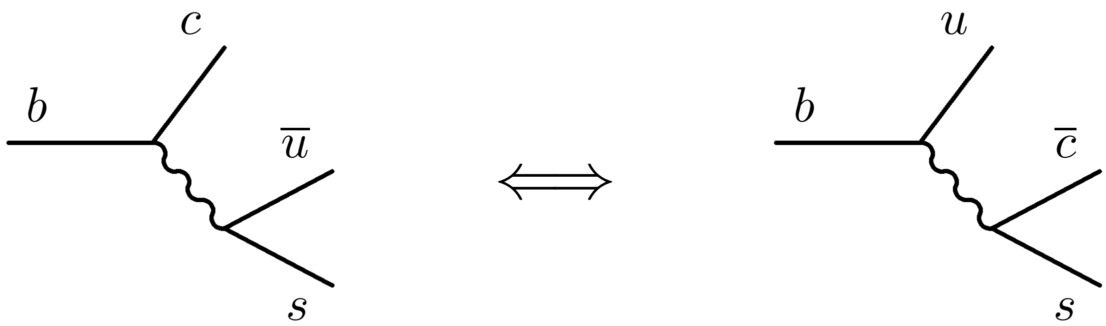
- $B^{\pm} \to K^{\pm}(K_S\pi^-\pi^+)_D$ cascade decay with a fit of Dalitz plot is a working method of measuring γ
- for future precision measurements a model independent version of the method exists

Backup slides

Obtaining γ

- use interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$

Gronau, Wyler, 1991



interference between

$B^- \to DK^-$	followed by	$D \to f$
$B^- \to \bar{D}K^-$	followed by	$\bar{D} \to f$

with f any common final state of D and $\bar{D}$

- no penguin contributions

Different methods

methods can be grouped by the choice of final state f

- CP- eigenstate (e.g. $K_S\pi^0$) Gronau, London, Wyler (1991)
- flavor state (e.g. $K^+\pi^-$) Atwood, Dunietz, Soni (1997)
- singly Cabibbo suppressed (e.g. $K^{*+}K^-$) Grossman, Ligeti, Soffer (2002)
- many-body final state (e.g. $K_S\pi^+\pi^-$) Giri, Grossman, Soffer, JZ (2003)

other extensions:

- many body B final states (e.g. $B^+ \to DK^+\pi^0$) Aleksan, Petersen, Soffer (2002)
- use D^{0*} in addition to D^0
- use self tagging D^{0**} Sinha (2004)
- neutral B decays (time dependent and time-integrated) many refs.

Comment on use of D^*

Bondar, Gershon (2004)

- define CP eigenstates

$$D^{(*)}_{CP=\pm} = \frac{1}{\sqrt{2}}\left(D^{0(*)} \pm \bar{D}^{0(*)}\right)$$

$$\left.\begin{array}{l}\text{CP}(\pi^0) = -1 \\ \text{CP}(\gamma) = +1\end{array}\right. \Rightarrow \qquad \begin{array}{l} D^*_\pm \to D_\pm \pi^0 \\ D^*_\pm \to D_\mp \gamma \end{array}$$

- introduces a sign flip

$$Br(B^\pm \to D^*[D_f\pi^0]K^\pm) = r_B^2 + r_D^2 + 2r_Br_D\cos(\delta_B + \delta_D \pm \gamma)$$

$$Br(B^\pm \to D^*[D_f\gamma]K^\pm) = r_B^2 + r_D^2 - 2r_Br_D\cos(\delta_B + \delta_D \pm \gamma)$$

$$A_D = r_D e^{-i\delta_D}\bar{A}_D$$

Decay width

reduced partial decay width

$$d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) = \Big(A^2_{12,13} + r_B^2\, A^2_{13,12}$$
$$+2r_B \mathcal{R}e\big[A_D(s_{12}, s_{13})\, A^*_D(s_{13}, s_{12})\, e^{-i(\delta_B-\gamma)}\big]\Big) dp$$

Master formulae- complete

$$\hat{\Gamma}_i^- \equiv \int_i d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) =$$

$$T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i],$$

$$\hat{\Gamma}_{\bar{i}}^- \equiv \int_{\bar{i}} d\hat{\Gamma}(B^- \to (K_S\pi^-\pi^+)_D K^-) =$$

$$T_{\bar{i}} + r_B^2 T_i + 2r_B[\cos(\delta_B - \gamma)c_i - \sin(\delta_B - \gamma)s_i],$$

$$\hat{\Gamma}_i^+ \equiv \int_i d\hat{\Gamma}(B^+ \to (K_S\pi^-\pi^+)_D K^+) =$$

$$T_{\bar{i}} + r_B^2 T_i + 2r_B[\cos(\delta_B + \gamma)c_i - \sin(\delta_B + \gamma)s_i],$$

$$\hat{\Gamma}_{\bar{i}}^+ \equiv \int_{\bar{i}} d\hat{\Gamma}(B^+ \to (K_S\pi^-\pi^+)_D K^+) =$$

$$T_i + r_B^2 T_{\bar{i}} + 2r_B[\cos(\delta_B + \gamma)c_i + \sin(\delta_B + \gamma)s_i].$$

Complete Eq. for CP viol. D

$$\int_i |A(B^- \to f_D K^-|^2 =$$

$$A_B^2 \Big\{ T_i + r_B^2 T_{\bar{i}} + 2 r_B [\cos(\delta_B - \gamma) c_i + \sin(\delta_B - \gamma) s_i]$$

$$- \tfrac{1}{2} \mathcal{R}e \Big[\big(\tfrac{q}{p} - \tfrac{p}{q}\big)^* (x + iy)^* \big\{ \big[c_i (1 + r_B^2) + r_B (T_{\bar{i}} - T_i) \cos(\delta_B - \gamma) \big]$$

$$+ i \big[s_i (1 - r_B^2) + r_B (T_{\bar{i}} + T_i) \sin(\delta_B - \gamma) \big] \big\}$$

$$+ 2 \sin\theta \mathcal{I}m \Big[\big(A_f^{\text{even}} + r_B e^{i(\delta_B - \gamma)} \bar{A}_f^{\text{even}} \big) \big(B_f - r_B e^{i(\delta_B - \gamma)} \bar{B}_f \big)^* \Big] \Big\}$$